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Journal of Economic Theory 163 (2016) 178–221

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Asymmetric all-pay contests with heterogeneous prizes ☆

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Received 29 September 2013; final version received 23 November 2015; accepted 10 December 2015

Available online 23 December 2015

Abstract

This paper studies complete-information, all-pay contests with asymmetric players competing for heterogeneous prizes. In these contests, each player chooses a performance level or "score". The first prize is awarded to the player with the highest score, the second – less valuable – prize to the player with the second highest score, etc. The players are asymmetric as they incur different scoring costs, and they are assumed to have ordered marginal costs. The prize sequence is assumed to be either geometric or quadratic. We show that each such contest has a unique Nash equilibrium, and we exhibit an algorithm that constructs the equilibrium. Then, we apply the results to study the issue of tracking in schools and the optimality of winner-take-all contests.

JEL classification: D44; D72

Keywords: All-pay; Asymmetric; Contest; Heterogeneous

☆ First draft: November 2011. I would like to thank Vijay Krishna for his guidance, Ethem Akyol, Filippo Balestrieri, Peter Bardsley, Kalyan Chatterjee, Yeon-Koo Che, Nisvan Erkal, Edward Green, Simon Loertscher, Claudio Mezzetti, Benny Moldovanu, John Morgan, Roberto Raimondo, Ron Siegel, Neil Wallace, the reading committee at the University of Melbourne and seminar participants for comments and discussion. Our editor and a group of referees greatly helped to improve the paper's content and exposition.

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http://dx.doi.org/10.1016/j.jet.2015.12.006

1. Introduction

Asymmetric players and heterogeneous prizes are predominant in contests. For example, students with different intelligence levels compete for different grades, athletes with different abilities compete for different medals, and employees with different experience compete for different promotion opportunities. The key characteristics common to these contests are: heterogeneous prizes awarded solely on the basis of relative performance; participants with possibly different abilities; and sunk costs of participants' investments.

Moreover, the prize sequences in such contests are usually convex – the difference between higher prizes is greater than the difference between lower ones. For instance, the difference in returns to higher education between top-ranked students and average students is typically much higher than the difference between the average students and low-ranked students. Convex prize structures are also common in sports. The winner of the 2013 US Open tennis tournament was awarded a prize of $2.6 million. The runner-up won $1.3 million whereas those in the joint third position – the losing semi-finalists – won $650 thousand each. The prize for a particular rank was roughly twice the prize for the next rank.[1]

This paper presents a contest model with the combination of asymmetric players and convex prize sequences. Specifically, we study complete-information all-pay contests in which participants with different abilities compete for heterogeneous prizes. The different abilities are represented by different costs of performance, and the marginal costs are ordered (a stronger participant's marginal cost is higher than that of a weaker participant at any performance level). The prize sequence is either *geometric* (the ratio of successive prizes is a constant, like at the US Open tennis tournament) or *quadratic* (the second-order differences are a positive constant). Each player chooses a costly performance level or "score". The player with the highest performance receives the highest prize, the player with the second-highest performance receives the second highest prize, and so on (the prizes may be allocated randomly in the case of a tie). A player's payoff is his winnings (if any) minus his cost of performance. The cost is incurred regardless of whether the player wins a prize or not.

Our main result is that such contests have a *unique* Nash equilibrium. Moreover, we provide an algorithm to construct the equilibrium. Both the uniqueness and construction rely on the algorithm. A key feature of the algorithm is that a weaker player's equilibrium payoff can be determined by examining his best response to the strategies in a smaller contest in which only players stronger than him participate. This feature, formally stated in Proposition 5, allows us to start with a set of stronger players and determine the equilibrium payoff of the next strongest player, and therefore derive his strategy. Then, we can move on to determine the equilibrium payoff of another, still weaker player.

These results allow us to tackle several challenges. If two or more participants have the same cost functions, there may be multiple equilibria. Moreover, prize allocation and total expected performance may differ across equilibria (see Example 2 below). In many applications, it is the planner's objective to maximize the total expected performance, so multiple equilibria make it challenging to compare different contest formats. In contrast, our result shows a unique equilibrium if no two participants have the same cost function. Therefore, the uniqueness of equilibrium

[1] Similarly, at the 2014 US Open golf tournament, the winner received $1.62 million, the two players tied for runner-up received $789 thousand each – the average of the prizes for positions 2 and 3, the five players tied for third-place received $326 thousand each – the average of the prizes for positions 4 to 8.