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Bounded computational capacity equilibrium ☆

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Abstract

A celebrated result of Abreu and Rubinstein (1988) states that in repeated games, when the players are restricted to playing strategies that can be implemented by finite automata and they have lexicographic preferences, the set of equilibrium payoffs is a strict subset of the set of feasible and individually rational payoffs. In this paper we explore the limitations of this result. We prove that if memory size is costly *and* players can use mixed automata, then a folk theorem obtains and the set of equilibrium payoffs is once again the set of feasible and individually rational payoffs. Our result emphasizes the role of memory cost and of mixing when players have bounded computational power.

JEL classification: C72; C73

Keywords: Bounded rationality; Automata; Complexity; Infinitely repeated games; Equilibrium

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1. Introduction

The literature on repeated games usually assumes that players have unlimited computational capacity or unbounded rationality. Since in practice this assumption does not hold, it is important to study whether and how its absence affects the predictions of the theory.

One common way of modeling players with bounded rationality is by restricting them to strategies that can be implemented by finite state machines, also called finite automata. The game theoretic literature on repeated games played by finite automata can be roughly divided into two categories. One backed by an extensive literature (e.g., Kalai, 1990, Ben Porath, 1993, Piccione, 1992, Piccione and Rubinstein, 1993, Neyman 1985, 1997, 1998, Neyman and Okada 1999, 2000a, 2000b, Zemel, 1989) that studies games where the memory size of the two players is determined exogenously, so that each player can deviate only to strategies with the given memory size. In the other, Rubinstein (1986), Abreu and Rubinstein (1988), and Banks and Sundaram (1990) study games where the players have lexicographic preferences: each player tries to maximize her payoff, and subject to that she tries to minimize her memory size. Thus, it is assumed that memory is free, and a player would deviate to a significantly more complex strategy if that would increase her profit by one cent. Abreu and Rubinstein (1988) proved that in this case, the set of equilibrium payoffs in two-player games is generally a strict subset of the set of feasible and individually rational payoffs. In fact, it is the set of feasible and individually rational payoffs that can be generated by a *coordinated play*; that is, a sequence of action pairs in which there is a one-to-one mapping between Player 1's actions and Player 2's actions. For example, in the Prisoner's Dilemma that appears in Fig. 1, where each player has two actions, C and D, this set is the union of the two line segments $(3,3)-(1,1)$ and $(3,1)-(1,3)$.

To obtain their result, Abreu and Rubinstein (1988) make two implicit assumptions: (a) memory is costless, and (b) players can use only pure automata. Removing assumption (a) while keeping assumption (b) does not change the set of equilibrium payoffs. Indeed, since the preference of the players is lexicographic, no player can profit by deviating to a larger automaton when memory is costless, so a fortiori she has no profitable deviation when memory is costly. The construction in Abreu and Rubinstein (1988) ensures that a deviation to a smaller automaton yields the deviator a payoff which is close to her min-max value in pure strategies. Therefore, as soon as memory cost is sufficiently small, there is no profitable deviation to a smaller memory as well. We do not know whether and how the set of equilibrium payoffs changes when removing assumption (b) and keeping assumption (a).

Our goal in this paper is to show that if one removes both assumptions (a) and (b), then the result of Abreu and Rubinstein (1988) fails to hold. We will show that if memory is costly (yet memory cost goes to 0) and players can use mixed strategies, then a folk theorem obtains, and the set of equilibrium payoffs includes the set of feasible and individually rational payoffs (w.r.t. the min-max value in pure strategies). We assume for simplicity that the players have additive utility: the utility of a player is the difference between her long-run average payoff and the cost of her computational power.

We thus present a new equilibrium concept that is relevant when memory size matters and each player's set of pure strategies is the set of finite automata. For a given positive real number c, we say that the vector $x \in \mathbb{R}^2$ is a *c-Bounded Computational Capacity equilibrium payoff* (hereafter, BCC for short) if it is an equilibrium payoff when the utility of each player is the difference between her long-run average payoff, and c times the size of its finite state machine.