Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 163 (2016) 410–434
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Robust option pricing: Hannan and Blackwell meet Black and Scholes ☆

Peter M. DeMarzo [a,*], Ilan Kremer [b,c], Yishay Mansour [d,e]

[a] *Stanford University, United States*
[b] *Hebrew University, Israel*
[c] *University of Warwick, United Kingdom*
[d] *Tel Aviv University, Israel*
[e] *Microsoft Research, Israel*

Received 1 September 2014; final version received 12 January 2016; accepted 18 January 2016

Available online 22 January 2016

Abstract

We apply methods developed in the literature initiated by Hannan and Blackwell on robust optimization, approachability and calibration, to price financial securities. Rather than focus on asymptotic performance, we show how gradient strategies developed to minimize asymptotic regret imply financial trading strategies that yield arbitrage-based bounds for option prices. These bounds are new and robust in that they do *not* depend on the continuity of the stock price process, complete markets, or an assumed pricing kernel. They depend only on the realized quadratic variation of the price process, which can be measured and, importantly, hedged in financial markets using existing securities. Our results also apply directly to a new class of options called timer options. Finally, we argue that the Hannan–Blackwell strategy is path dependent and therefore suboptimal with a finite horizon. We improve it by solving for the optimal path-independent strategy, and compare the resulting bounds with Black–Scholes.

JEL classification: C73; D81; G13

☆ We thank the editor, Alessandro Pavan, the associate editor, and several anonymous referees for their very helpful comments. We thank Sergiu Hart for numerous insightful discussions, as well as seminar participants at Berkeley (IEOR), Tel Aviv University, Hebrew University, Stanford, UCLA, Caltech, Duke, UIUC, University of Chicago, MIT and Washington University.

* Corresponding author.
E-mail address: pdemarzo@stanford.edu (P.M. DeMarzo).

http://dx.doi.org/10.1016/j.jet.2016.01.009
0022-0531/

Keywords: Approachability; Calibration; Regret minimization; Robust optimization; Option pricing; Arbitrage bounds

1. Introduction

There is a growing literature in game theory on calibration, expert testing, learning in games, and the dynamic foundations of correlated equilibria; see Hart (2005), Foster et al. (1999), Fudenberg and Levine (1998), and Cesa-Bianchi and Lugosi (2006) for excellent surveys.[1] This literature is based on earlier work by Hannan (1957) and Blackwell (1956) who studied robust dynamic optimization and defined "approachability" or "regret minimization" for games under uncertainty. Regret is defined as the difference between the outcome of a strategy and that of the *ex-post* optimal strategy (within a given class).[2]

Many of the results in this literature are truly remarkable. For example, in many settings they imply that an agent is able to dynamically optimize, in an uncertain and even non-stationary environment, in a way that mimics the asymptotic performance of an agent with full knowledge of the underlying distribution of uncertainty. But despite the elegance of such results, questions remain regarding the economic relevance of this asymptotic performance in a meaningful economic context with a finite horizon and discounted payoffs.

The goal of this paper is to bridge this gap by applying the regret minimization methodology to financial economics, as monetary payoffs provide a tangible way to measure the performance of regret-minimizing strategies that we believe is more meaningful than the standard approach of evaluating asymptotic average performance. In particular, we focus on standard European-style options, which we can think of as contracts that, in exchange for an upfront premium, allow investors to minimize their regret when choosing an investment portfolio. We then show how results based on approachability translate to upper bounds for option prices. These bounds are both new and empirically relevant; moreover, they depend only on the quadratic variation of the stock price process, a generalization of the usual measure of volatility which allows for jumps. Importantly, the quadratic variation is easily measurable and often contracted upon in practice. Thus, these bounds are relevant in a trading context as financial instruments exist to hedge against fluctuations in quadratic variation, and so we can obtain an arbitrage-based link between the price of options and the value of these instruments. The bounds we compute can also be directly applied to provide assumption-free bounds for a new class of options called 'timer options' that have become increasingly popular (see Section 4).

How does our approachability-based approach compare to traditional option pricing theory? Classic option pricing methods, such as the Black–Scholes–Merton or Binomial option pricing models, rely on highly restrictive assumptions regarding stock price paths to guarantee market completeness. The binomial model assumes stock prices move discretely with jumps that have a known magnitude, while the Black–Scholes framework assumes a continuous price process with a constant volatility. Indeed, these restrictions on the set of allowable price paths are violated by essentially *all* observed price paths in practice.

To get around this problem and deal with reality of market incompleteness, an alternative approach used in the theoretical option pricing literature is to assume a specific form for the

[1] Recent contributions include Dekel and Feinberg (2006), Al-Najjar and Weinstein (2008) and Olszewski and Sandroni (2008).

[2] We use the terms "regret minimization" and "approachability" interchangeably. Formally, approachability is a more general concept whereas regret minimization is a classic application of it.