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Continuous Markov equilibria with quasi-geometric discounting

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Abstract

This paper proves that the standard quasi-geometric discounting model used in dynamic consumer theory and political economics does not possess Markov perfect equilibria (MPE) with continuous decision rules, if there is a strictly positive lower bound on wealth. It is shown that these discontinuities imply that the decision-maker strictly prefers lotteries over next period's assets. An extension with lotteries is presented, and the existence of an MPE with continuous decision rule is established. The models with and without lotteries are numerically compared, and some appealing properties of the lottery-enhanced model are noted.

JEL classification: C73; D11; D90; E21; H63; P16

Keywords: Quasi-geometric; Quasi-hyperbolic; Markov perfect equilibrium; Debt limit; Continuous solutions; Lotteries

1. Introduction

Intertemporal preferences with quasi-geometric (or quasi-hyperbolic) discounting have been proposed for studying optimal national savings policy with imperfect altruism (Phelps and Pollak, 1968), for studying the savings behavior of individuals who value commitment to a consumption plan (Laibson, 1997) and for studying optimal growth and asset pricing outside of the straitjacket of geometric discounting (Barro, 1999; Luttmer and Mariotti, 2003). In addition,

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quasi-geometric discounting may arise endogenously in models of collective decision-making. Examples include models of political turnover and disagreement (Alesina and Tabellini, 1990 and Persson and Svensson, 1989), models of majority-based legislative decision-making (Battagliani and Coate, 2008), and nonunitary models of household decision-making (Hertzberg, 2012).[1]

Decision problems with quasi-geometric discounting must be analyzed as a dynamic game because of the implied time inconsistency of intertemporal preferences (Strotz, 1956). This paper is motivated, in part, by a puzzle regarding the computation of Markovian equilibria of this game. Consider a decision-maker (henceforth DM) with an infinite planning horizon, constant endowment, and facing a constant gross interest rate equal to the inverse of the (long-run) discount factor between any two future consecutive periods, β. The (short-run) discount factor between the current period and the next is $\delta\beta$ ($\delta < 1$). In this situation, the "present bias" introduced by δ should cause DMs to persistently dissave. Indeed, for models in which a closed-form Markovian solution can be found, the solution displays continuous (and smooth) dissaving behavior. However, when the equilibrium of the same model is computed on a grid using value (and policy) function iteration, the decision rule found has multiple points of discontinuity, and these points are typically stationary points as well (i.e., steady states with no dissaving). This is true even when the analytical solution is fed in as the initial guess.

The first contribution of this paper is to show that any Markovian decision rule of this stationary environment with CRRA preferences must be discontinuous, if the DM's net wealth (i.e. the present value of endowments less debt) cannot fall below a strictly positive value.[2] This has two implications: First, the known continuous (and smooth) analytical solutions for stationary environments work because the DM's wealth is allowed to get arbitrarily close to zero. Second, it explains why computed solutions feature discontinuities even when there is a known continuous and smooth analytical solution. Since value function iteration is done on a grid, the method imposes a de facto lower bound on wealth. Thus, the computation yields a discontinuous solution because the model being computed has discontinuous solutions only.[3]

The second contribution is motivated by the fact that a positive lower bound on wealth is a natural assumption in many applications, for instance, when the DM is an individual facing a borrowing constraint. The discontinuity of Markovian decision rules is, then, an intrinsic property of the equilibrium. However, it is shown that these discontinuities always reflect nonconcave segments of the continuation value function and, consequently, actuarially fair lotteries (over next period's asset choice) raise ex-ante welfare of the DM.[4] Based on this, an extension of the standard quasi-geometric model to lotteries is presented. The extension has several impor-

[1] Several recent models with political frictions (Aguiar and Amador, 2011 and Azzimonti, 2011) feature versions of "present-bias" characteristic of quasi-geometric discounting.

[2] Krusell and Smith (2003) present an algorithm for constructing a continuum of discontinuous Markovian decision rules for the neoclassical growth model. In the linear case, their construction requires that the gross interest rate strictly exceed $1/\beta$ and, thus, does not apply to the environment of this paper. Furthermore, they do not prove (or suggest) that discontinuities are a necessary feature of Markovian decision rules.

[3] In the past, the occurrence of discontinuous (computed) solutions when smooth solutions were expected was interpreted as an instance of multiple equilibria. Perhaps guided by this assessment, researchers have been content to restrict attention to parameter values for which discontinuous solutions ("pathologies") do not arise (Laibson et al., 1998) or have adopted methods other than finite-state value function iteration to locate smooth solutions (Krusell et al., 2002; Judd, 2004; Maliar and Maliar, 2005).

[4] The fact that time-inconsistent preferences may imply nonconcave value functions was shown in the working paper version of Luttmer and Mariotti (2007) for a three-period exchange economy with complete markets and no borrowing constraint (STICERD, Discussion Paper TE/03/446, January 2003). Since actuarially fair lotteries are equivalent to fair-value gambles (Cole and Prescott, 1997), this result points to an underappreciated implication of quasi-geometric