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Bounded memory Folk Theorem ☆

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Abstract

We show that the Folk Theorem holds for n-player discounted repeated games with bounded memory (recall) strategies. Our main result demonstrates that any payoff profile that exceeds the pure minmax payoff profile can be approximately sustained by a pure strategy finite memory subgame perfect equilibrium of the repeated game if the players are sufficiently patient. We also show that the result can be extended to any payoff profile that exceeds the *mixed* minmax payoff profile if players can randomize at each stage of the repeated game. Our results requires neither time-dependent strategies, nor public randomization, nor any communication. The type of strategies we employ to establish our result turn out to have new features that may be important in understanding repeated interactions.

JEL classification: C72; C73; C79

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1. Introduction

The extensive multiplicity of subgame perfect equilibrium (SPE) payoffs in repeated games, exemplified by the Folk Theorem, is due to players' ability to condition their behavior arbitrarily

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on the past (see Mailath and Samuelson, 2006). Therefore, it is reasonable to expect, as suggested by Aumann (1981), that this multiplicity may be reduced if players have limited memory in the sense that they can condition their strategies only on the outcome of a limited number of past periods.

In Barlo et al. (2009), we show that this intuition, however, does not hold when the set of actions in the stage game of the repeated game is sufficiently "large" so that each payoff profile is not isolated. In such games we prove that the Folk Theorem with SPE as the solution concept (henceforth, we shall refer to such Folk Theorems by FT) continues to hold with one period memory strategies where at each date players' behavior depends only on the outcome of the game in the previous period. The large action space assumption is critical in establishing this result because it allows players to encode the entire history of the past into the previous period's actions.

In the same study, we show that when the action spaces are not large, it is possible that no efficient payoff vector can be supported by a one period memory SPE strategy profile even if the discount factor is near one, validating the argument of Aumann (1981) with one period memory strategies and finite actions. Hence, the question is whether or not the multiplicity of equilibrium payoffs prevails with finite actions and limited memory (not necessarily restricted to be one period). More specifically, does the FT depend critically on being able to recall the history of play all the way back to the beginning?

The current paper establishes that the FT for discounted repeated games continues to hold with time-independent bounded memory strategies even when the action sets are finite. Our main result displays that, when players are sufficiently patient, any feasible payoff vector that guarantees each player at least his pure strategy minmax payoff (individually rational payoffs) can be approximately sustained by a pure SPE strategy profile of the repeated game that at each stage recalls the outcomes of finite number of previous periods.[1] Furthermore, we show that the bound on the number of periods that the players need to recall to establish this result is uniform in the level of discounting, and depends only on the desired degree of payoff approximation.

With no memory restriction, the FT result with mixed (behavioral) strategies is stronger than that with pure strategies. This is because for any player the mixed strategy minmax payoff may be lower than the pure strategy minmax payoff. To complete the analysis of repeated games with finite memory, we extend our result for pure strategies to show that with three or more players, if players are sufficiently patient and are allowed to use behavioral strategies, then any payoff vector that guarantees each player at least his mixed minmax payoff profile can be approximately sustained by a behavioral SPE strategy profile that at each stage recalls the outcomes of finite number of previous periods. The following points need to be emphasized regarding our finite memory mixed FT result: First, it assumes that the players observe only the outcome of past randomizations (and not the randomization devices used in the past).[2] Second, it is obtained without introducing any public randomization or any external communication devices. Third, in contrast to our pure strategy result, the bound on the number of periods that the players need to recall to establish the mixed FT is not uniform in the level of discounting.

[1] As it is the case for Fudenberg and Maskin's (1986) FT results, our FT result with more than 2 players is established for generic games.

[2] If randomization devices employed in the past are observable then the repeated game with mixed strategies is equivalent to one with a continuum of action space at each stage. Hence, it follows from Theorem 9 of Barlo et al. (2009) that the mixed FT holds with 1-period memory.