Available online at www.sciencedirect.com

ScienceDirect

Journal of Economic Theory 163 (2016) 826–848

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Common belief foundations of global games ☆

Stephen Morris [a,*], Hyun Song Shin [b], Muhamet Yildiz [c]

[a] *Princeton University, United States*
[b] *Bank for International Settlements, Switzerland*
[c] *M.I.T., United States*

Received 6 January 2016; final version received 27 February 2016; accepted 14 March 2016

Available online 18 March 2016

Abstract

We study coordination games under general type spaces. We characterize rationalizable actions in terms of the properties of the belief hierarchies and show that there is a unique rationalizable action played whenever there is approximate common certainty of rank beliefs, defined as the probability the players assign to their payoff parameters being higher than their opponents'. We argue that this is the driving force behind selection results for the specific type spaces in the global games literature.

JEL classification: C72; D82; D84

Keywords: Global games; Risk-dominance; Rank beliefs; Higher-order beliefs; Rationalizability

1. Introduction

Complete information games often have many equilibria. Even when they have a single equilibrium, they often have many actions that are rationalizable, and are therefore consistent with

☆ This paper incorporates material from a working paper of the same title circulated in 2007 (Morris and Shin, 2007). The first two authors are grateful to the National Science Foundation Grant SES-0648806 for funding this research. We are grateful for comments from editors, anonymous referees, and seminar participants at Columbia, Iowa State, Northwestern, and UCLA on this iteration of the project. We thank Anton Tsoy for detailed comments. The views expressed in this paper reflect those of the authors and not necessarily those of the Bank for International Settlements.

* Corresponding author.
E-mail address: smorris@princeton.edu (S. Morris).

http://dx.doi.org/10.1016/j.jet.2016.03.007

common certainty of rationality. The inability of theory to make a prediction is problematic for economic applications of game theory.

Carlsson and van Damme (1993) suggested a natural perturbation of complete information that gives rise to a unique rationalizable action for each player. They introduced the idea of "global games"—where any payoffs of the game are possible and each player observes the true payoffs of the game with a small amount of noise. They showed—for the case of two player two action games—that as the noise about payoffs becomes small, there is a unique equilibrium; moreover, the perturbation selects a particular equilibrium (the *risk-dominant* one) of the underlying game. This result has since been generalized in a number of directions and widely used in applications.[1] When the global game approach can be applied to more general games, it can be used to derive unique predictions in settings where the underlying complete information game has multiple equilibria, making it possible to carry out comparative static and policy analysis. It has been informally argued that multiplicity partly relies on the unrealistic "complete information" assumption, and the natural perturbation underlying global games captures the more realistic case.

However, the global game selection result uses a particular form of perturbation away from "complete information." Complete information entails the assumption that a player is certain of the payoffs of the game, certain that other players are certain, and so on. Weinstein and Yildiz (2007) consider more general perturbations, saying that a situation is close to a complete information game if players are almost certain that payoffs are close to those complete information game payoffs, almost certain that other players are almost certain that payoffs are close to those payoffs, and so on. Formally, they consider closeness in the product topology on the universal belief space. They show that for *any* action which is rationalizable for a player in a complete information game, there exists a nearby type of that player in the product topology for whom this is the unique rationalizable action. Thus by considering a richer but also intuitive class of perturbations, they replicate the global game uniqueness result but reverse the selection result.

In this paper, we identify the driving force behind global game uniqueness and selection results. In particular, we do not want to take literally the (implicit) assumption in global games that there is common certainty among the players of a technology which generates (conditionally independent) noisy signals observed by the players. Rather, we want to argue that global game perturbations are a metaphor, or a convenient modeling device, for a more general intuitive class of relaxations of common certainty. We want to characterize and analyze the key property of that more general class, which must also be more restrictive than the product topology perturbations of Weinstein and Yildiz (2007).

Our baseline analysis is carried out for a two player, two action game. Each player must decide whether to "invest" or "not invest". Payoffs are given by the following matrix:

	invest	not invest
invest	$x_1,\ x_2$	$x_1 - 1,\ 0$
not invest	$0,\ x_2 - 1$	$0,\ 0$

(1)

[1] Morris and Shin (1998) analyzed a global game with a continuum of players making binary choices, and this case has been studied in a number of later applications. See Morris and Shin (2003a) for an early survey of some theory and applications of global games. Frankel et al. (2003) study global game selection in general games with strategic complementarities.