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Ordinal Bayesian incentive compatibility in restricted domains ☆

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Abstract

We study deterministic voting mechanisms by considering an ordinal notion of Bayesian incentive compatibility (OBIC). If the beliefs of agents are independent and generic, we show that a mechanism is OBIC and satisfies an additional condition called *elementary monotonicity* if and only if it is a dominant strategy incentive compatible mechanism. Our result works in a large class of preference domains (that include the unrestricted domain, the single-peaked domain, the single-dipped domain, and some single-crossing domains). We can significantly weaken elementary monotonicity in our result in the single-peaked domain if we assume unanimity and in a large class of domains if we assume unanimity and tops-onlyness.

JEL classification: D71; D82

Keywords: Ordinal Bayesian incentive compatibility; Single-peaked domain; Elementary monotonicity

1. Introduction

In standard models of voting, dominant strategy incentive compatibility (DSIC) is usually too demanding. This is illustrated by the Gibbard–Satterthwaite theorem (Gibbard, 1973;

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Satterthwaite, 1975), which shows that the only DSIC and unanimous deterministic voting mechanism in the unrestricted domain is a dictatorship.[1] This motivates the study of weaker solution concepts in these models. In this paper, we consider ordinal Bayesian incentive compatibility (OBIC) introduced by d'Aspremont and Peleg (1988). A voting mechanism is OBIC if for every agent, his interim/expected outcome probability vector from truth-telling first-order stochastic-dominates any interim outcome probability vector obtained by deviating. In the unrestricted domain of preferences, Majumdar and Sen (2004) show that OBIC with independent and generic priors is equivalent to DSIC under unanimity.[2] We investigate the robustness of this result to the unrestricted domain assumption.

We construct a *non-DSIC*, unanimous, and *anonymous* mechanism that is OBIC with respect to some generic priors when the domain of preferences is restricted to be the single-peaked domain. However, our main results suggest that the equivalence between OBIC and DSIC voting mechanisms can be restored in various restricted domains under weak additional axioms. The main additional axioms that we use are *elementary monotonicity* and its weaker versions along with unanimity. Elementary monotonicity, which we formally define later, is a very mild form of Maskin monotonicity, and requires a mechanism to respond *positively* to changes in the preferences of agents. It is satisfied by a variety of mechanisms.

Our core result says that OBIC and elementary monotonicity are equivalent to DSIC in a large class of domains. In the single-peaked domain, the equivalence between OBIC and DSIC holds with a significantly weaker version of elementary monotonicity if we assume unanimity. If we assume unanimity and *tops-onlyness*, the weakened version of elementary monotonicity and OBIC are equivalent to DSIC in a large class of domains.

Our results provide a foundation for using dominant strategy voting mechanisms in various restricted domains if we use ordinal deterministic mechanisms.[3] An implication of our results is that if we want to design Bayesian incentive compatible voting mechanisms, we must consider randomized and/or cardinal mechanisms. All our results hold even if we weaken OBIC to only prevent manipulations of each agent to his *adjacent* preferences – we call this requirement *locally* OBIC (LOBIC). Incentive compatibility with local incentive constraints were recently studied in Carroll (2012) and Sato (2013), who identified domains where local incentive constraints imply all incentive constraints. All our proofs use ideas from this literature. Thus, our results bring together two different ideas (OBIC and local incentive compatibility) in strategic voting literature.

Our results extend the result in Majumdar and Sen (2004) by identifying the precise connection between DSIC and OBIC mechanisms with (and without) unanimity in restricted domains. Also, they corroborate the different implications of OBIC with generic and uniform priors (a uniform prior requires that each preference in the domain is drawn with equal probability). This is because Majumdar and Sen (2004) had shown that every *neutral* mechanism satisfying elementary monotonicity is OBIC under uniform priors in the unrestricted domain – this covers many reasonable mechanisms. In contrast, our results show a very different implication of elementary monotonicity with generic priors.

[1] Throughout the paper, we only consider deterministic voting mechanisms.

[2] We define a generic prior formally later – it is a generic subset of the set of independent priors.

[3] Restricting attention to ordinal and deterministic mechanisms in this setting is with loss of generality – see Borgors and Postl (2009), Borgors and Smith (2014), who discuss this issue in detail.