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Lexicographic beliefs and assumption ☆

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Abstract

Foundations for iterated admissibility (i.e., the iterated removal of weakly dominated strategies) need to confront a fundamental challenge. On the one hand, admissibility requires that a player consider every strategy of their opponents possible. On the other hand, reasoning that the opponents are rational requires ruling out certain strategies. Brandenburger, Friedenberg, Keisler's (BFK, *Econometrica*, 2008) foundations for iterated admissibility address this challenge with two ingredients: lexicographic beliefs and the concept of "assumption." However, BFK restrict attention to lexicographic beliefs whose supports are essentially disjoint. This restriction does not have a compelling behavioral rationale, or a clear intuitive interpretation. At the same time, it plays a crucial role in BFK's foundations for iterated admissibility—specifically, in their analysis of assumption. We provide an alternate characterization of assumption, which applies to all lexicographic beliefs. We also characterize two variants of assumption, based on two extensions of 'weak dominance' to infinite state spaces. These notions of assumption coincide with BFK's notion when the state space is finite and lexicographic beliefs have disjoint support; but they are different in more general settings. Leveraging these characterization results, we show that disjoint supports do not play a role in the foundations for iterated admissibility.

JEL classification: C72; D81; D83

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1. Introduction

Lexicographic beliefs (henceforth ℓ-beliefs) have become a relatively standard tool, both for studying refinements and for providing epistemic characterizations of solution concepts.[1] The appeal of ℓ-beliefs is that they can be used to address a tension between being certain that an opponent is rational and having full-support beliefs about opponents' actions. To clarify, suppose that, if Bob is rational, he will not play specific actions. Can Ann be certain that Bob is rational, and at the same time be cautious and assign non-zero probability to all of Bob's actions? The answer is no if Ann has standard probabilistic beliefs. Suppose instead that Ann has ℓ-beliefs. That is, she has a vector $(\mu_0, \dots, \mu_{n-1})$ of probabilities over the relevant space of uncertainty, S_b (Bob's strategy space) and uses them lexicographically to determine her preferences over her own strategies: Ann first ranks her strategies using μ_0; if that leads to more than one best reply for Ann, she uses μ_1 to rank them, and so on. If the union of the supports of the probabilities μ_i is all of S_b, then Ann's beliefs have, in a sense, full support. At the same time, Ann can still be confident in Bob's rationality, for example in the sense that the primary hypothesis μ_0 assigns positive probability only to strategies of Bob that are rational.

There are two notions of ℓ-beliefs that have been studied and used in the literature: lexicographic conditional probability systems (henceforth LCPSs) in which, loosely speaking, the supports of the different beliefs (i.e., the μ_i's) are disjoint, and the more general class of lexicographic probability systems (LPSs) in which this disjointedness condition is not imposed. In particular, LCPSs are used by Brandenburger et al. (2008, henceforth, BFK) to provide an epistemic characterization of iterated admissibility—thereby answering a long-standing open question.[2]

However, there are reasons not to find the restriction to LCPSs appealing. First, while Blume et al. (1991b) provide an axiom that characterizes LCPSs within the class of LPSs, their axiom has a flavor of reverse-engineering: it says no more than the probabilities in the LPS have disjoint support; it offers no further normative or other appeal. Indeed, the interpretation of LPSs is quite natural and intuitive. The probability μ_0 is the player's primary hypothesis, in the sense that she is (almost fully) confident in it. The probability μ_1 is her secondary hypothesis: she is willing to entertain it as an alternative, but considers it "infinitely" less plausible than μ_0; and so on. There is no reason that primary and secondary hypotheses must have disjoint supports. For instance, one may be confident that a coin is fair, but entertain the secondary hypothesis that it is biased towards falling on heads.[3] Second, the marginal of an LCPS need not be an LCPS. For example, suppose that two players are playing the game in Fig. 1, where the pairs of actions A, B for

[1] See, for example, Blume et al. (1991a), Brandenburger (1992), Stahl (1995), Mailath et al. (1997), Rajan (1998), Asheim (2002), Govindan and Klumpp (2003), Brandenburger et al. (2008), Keisler and Lee (2010), Lee (2016), Yang (2015), and Catonini and De Vito (2014) amongst many others.

[2] To be more precise: BFK provide an epistemic characterization of m rounds of deleting inadmissible strategies, for any finite m. Their epistemic conditions involve finite-order reasoning. However, they show an "impossibility result" for common reasoning—that is, common reasoning is impossible in their model.

[3] Of course one may instead have the secondary hypothesis that the coin will fall on an edge, which would have disjoint support, but that does not seem like the only story one could tell.