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Strategy-proof house allocation with price restrictions

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Abstract

This paper considers a house allocation problem with no initial ownership and where prices are bounded from below and above by exogenously given price restrictions. This type of housing market contains, e.g., the "assignment market" and the "student placement problem" as special cases. A mechanism called the minimal RPE mechanism is defined, and two main results are obtained. First, it is demonstrated that the mechanism is manipulable at some profile in the full preference domain $\mathcal{R}$. Second, it is proved that there is a subset $\tilde{\mathcal{R}} \subset \mathcal{R}$ of the full domain, containing almost all profiles in $\mathcal{R}$, such that the minimal RPE mechanism is strategy-proof in that subset.

JEL classification: C78; D71

Keywords: House allocation; Matching; Price restrictions; Strategy-proofness; Domain restriction

1. Introduction

We consider a house allocation problem with no initial ownership where prices are bounded from below and above by exogenously given price restrictions. Because the houses are indivisible objects, the upper and lower price restrictions are allowed to coincide, and the upper and lower price restrictions can take any real number, this problem contains, e.g., the models by Balinski and Sönmez (1999), Demange and Gale (1985), Hylland and Zeckhauser (1979), Leonard (1983), and Shapley and Shubik (1972), as special cases. Our main objective is to define a strategy-proof

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allocation mechanism for this type of housing market.[1] One difficulty in achieving this task is that, due to the exogenously given price constraints, a price equilibrium needs not exist for all preference profiles in the full preference domain (i.e., all profiles where preferences are rational, monotonic, and continuous). Consequently, previous results, from, e.g., Demange and Gale (1985) and Leonard (1983), cannot be used directly to construct a strategy-proof allocation mechanism. However, as is demonstrated in this paper, the basic idea in those papers, i.e., that a minimal price vector can be used as a main ingredient in a strategy-proof allocation mechanism, can also be used in non-equilibrium situations.

Since the price restrictions exclude equilibrium for certain preference profiles, the concept of a rationing price equilibrium (RPE, henceforth) is introduced. This concept is partly characterized by a priority-order that is used to determine the allocation when no price equilibrium exists, and partly characterized by a price condition stating that prices on "overdemanded" houses must equal the upper price bound. The investigated allocation mechanism then selects an RPE with a price vector that is minimal in the set of RPE price vectors. This mechanism is called a minimal RPE mechanism.

The first main insight of this paper is negative. More explicitly, it is shown that a minimal RPE mechanism is manipulable at some profile in the full preference domain $\mathcal{R}$. This negative finding provides a rationale for restricting the preference domain, and it also adds another example to the long line of examples that demonstrate that allocation mechanisms or social choice rules often need to be defined on restricted domains (e.g., single-peaked domains, or domains where the preferences satisfy the single-crossing condition or the intermediateness condition, etc.) to avoid impossibility results.[2]

The second main result of this paper is positive and demonstrates that there is a subset $\tilde{\mathcal{R}} \subset \mathcal{R}$ of the full domain, containing, in a mathematical meaning, almost all profiles in $\mathcal{R}$, such that the minimal RPE mechanism is strategy-proof in that subset. This result extends a partial result obtained for the minimal RPE mechanism in Andersson and Svensson (2014). In that paper, the mechanism was defined only on the subset $\tilde{\mathcal{R}}$, and it was shown that it is impossible for any group of agents to manipulate the mechanism if they report preferences restricted to profiles in $\tilde{\mathcal{R}}$. Hence, the result did not reveal if it is possible to manipulate the mechanism with a profile in $\mathcal{R} \setminus \tilde{\mathcal{R}}$. In this meaning, strategy-proofness was not proved. This in combination with the fact that the domain $\tilde{\mathcal{R}}$ is not a hyperrectangle is problematic as the reports of the truthful agents *and* the misrepresenting agents jointly determine whether or not the "new" preference profile belongs to the restricted domain $\tilde{\mathcal{R}}$, meaning that it is not possible for a group of agents to know that it is impossible to manipulate until they know the reported preferences of the truthful agents. As the main result of this paper demonstrates that the minimal RPE mechanism in fact is strategy-proof, the agents need not to have any information about other agents' preferences, or, put differently, the "new" profile is always allowed to belong to the rectangular domain $\mathcal{R}$.

The remaining part of this paper is organized as follows. Section 2 describes the basic economy and introduces a number of definitions and remarks. All results of this paper are stated and proved in Section 3. Appendix A contains a measure on sets of profiles such that $\tilde{\mathcal{R}} = \mathcal{R}$ a.e.

[1] Some recent papers have considered a housing market with price restrictions, see, e.g., Andersson et al. (2015), Herings (2015), Talman and Yang (2008) or Zhu and Zhang (2011), but none of these papers address the issue of strategy-proofness.

[2] See, e.g., Barberà et al. (2013) for a nice overview of domain restrictions and their consequences.