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A test for risk-averse expected utility ☆

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Abstract

We provide a universal condition for rationalizability by risk-averse expected utility preference in a demand-based framework with multiple commodities. Our test can be viewed as a natural counterpart of a classical test of expected utility, due to Fishburn (1975), in a demand setting.

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1. Introduction

The recent contribution of Kubler et al. (2014) provides a GARP-like test for risk-averse expected utility maximization in a contingent-consumption environment. In an environment with a single consumption good and finite states of the world, they establish an acyclicity condition on observed data which is both necessary and sufficient for a finite list of observed price and consumption pairs to be consistent with the hypothesis of expected utility maximization. Thus, their paper provides a counterpart of the classical work of Afriat (1967) with the added restriction that rationalizations be risk-averse expected utility.

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As Kubler et al. (2014) note, their test is universal in nature, removing all existential quantification. Their test amounts to verifying that the product of certain cycles of risk-neutral prices be bounded above by one. Our aim in this note is to provide a *different* universal test. Our test should be distinguished from the Kubler et al. (2014) test in three ways. First, it applies to any finite number of consumption goods, whereas the test of Kubler et al. (2014) only applies for a single consumption good. Secondly, our test is intimately tied to the classical von Neumann–Morgenstern axioms of expected utility theory, and thus has a simple economic intuition. On the other hand, our test involves universal quantification over a potentially infinite number of objects, while the test in Kubler et al. (2014) can be reduced to universal quantification over a finite set.

We emphasize that what we mean by *test* is a method for falsifying the model with directly observable data. In other words, we say a model is *testable* if whenever data are inconsistent with the model, they can be demonstrated to be inconsistent. In this sense of the term test, a demonstration is distinct from an algorithm which would find this falsifying certificate. Hence, a test in our sense is not intended to be useful from a computational perspective, and as far as we can tell, ours is not in general. Indeed; there are already practical algorithms for determining when the expected utility model is falsified in our context. Rather, such a test is important for understanding the economic content of the model, by specifying a condition stated in terms of data alone, which does not reference unobservable concepts such as utilities or marginal rates of substitution. As a point of comparison, the work of Richter (1966) can be understood as providing the testable restrictions of the preference maximization hypothesis; however, no general algorithm would exist in Richter's case either.[1]

Our test is perhaps most closely related to an early revealed preference test of expected utility due to Fishburn (1975). Fishburn constructs a test for an abstract environment of choice over lotteries with finite support. In his setting, one observes a finite set of binary comparisons; some are weak, and some are strict. Fishburn provides necessary and sufficient conditions for there to exist an expected utility ranking which extends the observed binary comparisons. Imagine that we observe lottery l_k weakly preferred to lottery l'_k for $k = 1, \ldots, g$, and l_k strictly preferred to l'_k for $k = g + 1, \ldots, K$. Fishburn establishes that these observations are consistent with expected utility maximization if there is no probability distribution over $\{1, \ldots, K\}$ which puts positive probability on $\{g + 1, \ldots, K\}$, and for which the mixture of the l_k's under this probability distribution is equal to the mixture of the l'_k's. Fishburn's test can be viewed as claiming that the smallest possible extension of the observed relations satisfying both independence and transitivity leads to no contradiction. We stress that Fishburn's test also presents with no algorithm: no recipe is given for finding the probability distribution.

In our case, we have n commodities, and a finite set of states $\Omega = \{\omega | 1, 2 \ldots, S\}$. We observe a finite list of prices and contingent consumption bundles chosen at those prices (x^k, p^k), $k \in \{1, \ldots K\}$. Consumption in state ω at observation k is of the form $x^k_\omega \in \mathbb{R}^n_+$. Probabilities over Ω are known and are given by the full support distribution π.

We first ask: What could reveal a violation of the joint hypothesis of expected utility and risk aversion in this context? There are only a finite set of states of the world, with known probabilities, but if the choices *were* rationalizable by an expected utility preference, there would be a natural extension to a preference over the set of all simple lotteries. One such violation would look like the following: suppose that for each x^k, there is some y^k which is feasible at prices p^k.

[1] In the special case where budgets are given by linear inequalities and preference satisfies monotonicity, an algorithm exists for Richter's test, namely the Afriat test. Here we refer to the abstract budget environment.