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# Optimal dynamic mechanism design with deadlines ☆

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**Abstract**

A seller maximizes revenue from selling an object in a dynamic environment, with buyers that differ in their patience: Each buyer has a privately known deadline for buying and a privately known valuation. First, we derive the optimal mechanism, neglecting the incentive constraint for the deadline. The deadline of the winner determines the time of the allocation and therefore also the amount of information available to the seller when he decides whether to allocate to a buyer. Depending on the shape of the markup that the seller uses, this can lead to a violation of the neglected incentive constraint. We give sufficient conditions on the type distribution under which the neglected constraint is fulfilled or violated. Second, for the case that the constraint cannot be neglected, we consider a model with two periods and two buyers. Here, the optimal mechanism is implemented by a fixed price in period one and an asymmetric auction in period two. The asymmetry, which is introduced to prevent the patient type of the first buyer from buying in period one leads to pooling of deadlines at the top of the type space.


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## 1. Introduction

In many situations, sellers face a changing population of heterogeneous buyers. Buyers arrive at different points in time. Some are impatient and want to buy immediately, others are patient and willing to wait. Patient buyers can act strategically and use their flexibility with respect to the time of a purchase in order to get better prices. Typical examples are online auctions, the sale of flight tickets, hotel reservations, or the sale of houses.

To capture heterogeneity in the degree of patience, we assume that buyers have idiosyncratic deadlines. A deadline can be viewed as an extreme form of time preferences, as in the case of a traveler who needs to buy tickets before a certain date in order to coordinate with other travel arrangements. Deadlines may also be imposed by third parties. Consider for example a company that needs to buy an input from a seller in order to enter a contractual relationship with a third party. This input could be a physical object, an option contract, a license, a patent, etc. It is conceivable that the third party sets a deadline, after which the contractual relationship is no longer available. Therefore, the input is worthless for the company if it is purchased after the deadline.

This paper analyzes the implications of private information about patience (deadlines) on the revenue maximizing mechanism. We consider the allocation of a single object over a finite time horizon with randomly arriving buyers and independent private values. To focus on the effects of private information about time preferences, we assume that arrival times are observable for the seller.[1] Consumption is assumed to take place at the end of the time horizon (e.g., when the plane takes off and not when the ticket is sold), so that we can ignore discounting as long as all discount rates are identical.

If we relax the incentive constraint for the deadline, revenue maximization is equivalent to maximizing expected virtual surplus, subject to monotonicity with respect to the valuation. Virtual surplus maximization is a straightforward dynamic programming problem which yields the *relaxed solution*. Since it is costless to delay an allocation and waiting for more buyers to arrive can improve revenue, the object is allocated only at the deadline of the winner. We show that the relaxed solution can be implemented by a payment rule that applies a markup to the *critical virtual valuation* of the winning buyer.[2] The latter is the lowest virtual valuation with which a buyer can win, given the types of all competing buyers and the seller's expectation of future arrivals. It can be interpreted as the *virtual opportunity cost* of awarding to the winning buyer.

The first contribution of the paper is a regularity condition under which the relaxed solution satisfies the neglected incentive constraint for the deadline.[3] We show a martingale property: Increasing the deadline leads to a mean-preserving spread of the virtual opportunity cost. Intuitively, a later deadline allows the seller to gather more information before allocating to a buyer, which increases the dispersion of the virtual opportunity cost. The martingale property is important, because for most distributions, the seller's markup is non-linear which induces endogenous risk-preferences in the buyers. If the markup is convex, buyers become endogenously risk-averse

[1] The case of unobservable arrivals is considered in Pai and Vohra (2013).

[2] Payoff equivalence implies that our results apply for any implementation of the relaxed solution.

[3] For simplicity we consider the case that the valuation and deadline of a buyer are independent. Section 3.4 discusses the correlated case.