# Collective rationality and monotone path division rules

## John E. Stovall

*Department of Economics, University of Warwick, Gibbet Hill Road, Coventry, CV4 7AL, United Kingdom*

## Abstract

We impose the axiom Independence of Irrelevant Alternatives on division rules for the conflicting claims problem. With the addition of Consistency and Resource Monotonicity, this characterizes a family of rules which can be described in three different but intuitive ways. First, a rule is identified with a fixed monotone path in the space of awards, and for a given claims vector, the path of awards for that claims vector is simply the monotone path truncated by the claims vector. Second, a rule is identified with a set of parametric functions indexed by the claimants, and for a given claims problem, each claimant receives the value of his parametric function at a common parameter value, but truncated by his claim. Third, a rule is identified with an additively separable, strictly concave social welfare function, and for a given claims problem, the amount awarded is the maximizer of the social welfare function subject to the constraint of choosing a feasible award.

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## 1. Introduction

A conflicting claims problem is a situation in which a divisible homogeneous good must be distributed among a group, each individual in the group having an objective claim on the good,

*E-mail address:* J.Stovall@warwick.ac.uk.

but where the amount of the good is insufficient to satisfy all the claims.[1] An example is dividing the liquidated value of a bankrupt firm among its creditors. How should the good be divided among the claimants? We seek a rule which chooses, for any problem, a feasible allocation or award. An award is feasible if it completely exhausts the good to be divided and if every individual receives an amount between 0 and his respective claim.

## 1.1. Overview of results

We impose the axiom Independence of Irrelevant Alternatives (IIA) on rules. This axiom states that if the chosen award for a problem is also feasible for a second problem whose feasible set is a subset of the original problem, then that award is also chosen for the second problem. This is the same axiom introduced by Nash [14] in the domain of bargaining problems. In the context of individual choice, this axiom is sometimes known as Chernoff's condition [3] or Sen's $\alpha$ [18].

We also impose two axioms that are common in the literature: Consistency and Resource Monotonicity. Consistency states that if a division rule chooses an award for a group of claimants, then it should not choose to reallocate the awards of any subgroup when considered as a separate problem. Resource Monotonicity states that if the amount to be divided increases, then no claimant's award should decrease.

Theorem 1 shows that IIA, Consistency, and Resource Monotonicity characterize a family of rules which can be described in three different but intuitive ways:

- Consider a fixed, weakly monotone path in the space of awards. For any group of claimants and any vector of claims for that group, the path of awards is simply the fixed path truncated by the claims vector. We refer to all such rules as monotone path rules.
- Consider a set of parametric functions, one for each individual. Each parametric function depends only on a single parameter, in which it is weakly increasing. For any problem, each parametric function is truncated by the individual's claim, and a common parameter is found so that the sum of the truncated parametric functions evaluated at that parameter equals the amount to be divided. We refer to all such rules as claims independent parametric rules.
- Consider an additively separable, strictly concave social welfare function. For any problem, the amount awarded is the maximizer of the social welfare function subject to the constraint of choosing a feasible award. We refer to all such rules as collectively rational additively separable (CRAS) rules.

We also consider a property which is dual to IIA. Rather than taking the awards as what matters to the individuals, as IIA does, this dual property takes the losses (the difference between an individuals claim and his award) as what matters. Theorem 2 shows that IIA and its dual are effectively incompatible: the queueing rule (which is generally considered to be normatively unappealing) is the only rule to satisfy Consistency, Resource Monotonicity, IIA, and the dual of IIA.

If there is no a priori reason to treat the claimants differently, then one would want the rule to give the same award to individuals with the same claim, a property known as Symmetry. Theorem 3 shows that the constrained equal awards rule is the only rule in our family which satisfies Symmetry.

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[1]  Though such problems are as old as civilization (examples are found in the Talmud), formal study of such problems began with O'Neill [16]. See Thomson [20,21] for surveys of the literature stemming from this seminal paper.