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On cooperative solutions of a generalized assignment game: Limit theorems to the set of competitive equilibria

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Abstract

We study two cooperative solutions of a market with indivisible goods modeled as a generalized assignment game: Set-wise stability and Core. We establish that the Set-wise stable set is contained in the Core and contains the non-empty set of competitive equilibrium payoffs. We then state and prove three limit results for replicated markets. First, the Set-wise stable set of a two-fold replicated market already coincides with the set of competitive equilibrium payoffs. Second, the sequence of Cores of replicated markets converges to the set of competitive equilibrium payoffs when the number of replicas tends to infinity. Third, for any number of replicas there is a market with a Core payoff that is not a competitive equilibrium payoff.

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1. Introduction

We study two cooperative solutions for a class of markets with indivisible goods modeled as generalized assignment games. Shapley and Shubik [11] defined an assignment game as a market where each seller owns one indivisible object and each buyer, who wants to buy at most one object, has valuations over all objects. An assignment is a description of deliveries of objects from sellers to buyers and a price vector is a list of prices, one for each object. A competitive equilibrium of a market is a price vector and a feasible assignment at which each seller maximizes revenues, each buyer maximizes net valuations, and markets clear. Shapley and Shubik [11] showed that the set of competitive equilibria is non-empty, competitive equilibrium assignments are optimal (the first welfare theorem holds), any optimal assignment is part of a competitive equilibrium with any of the competitive equilibrium price vectors (a strong version of the second welfare theorem holds without requiring any redistribution of the initial endowments), and the set of competitive equilibrium payoffs coincides with the Core of a naturally associated TU-game (no enlargement or replica of the market is required for their coincidence).

We consider a generalized assignment game representing a market with a given number of indivisible units of different goods, where sellers may own different units of each of the goods and buyers, who may want to buy several units of different goods up to an exogenous total amount, have constant marginal valuations of each good. Jaume, Massó, and Neme [5] extend Shapley and Shubik's [11] results for this generalized assignment game. In particular, they show that the set of competitive equilibria is non-empty, it is the Cartesian product of the set of competitive equilibrium price vectors and the set of optimal assignments, the set of competitive equilibrium price vectors has a lattice structure with the natural partial order of vectors $\geq$ "to be larger or equal than", and this lattice structure is partly translated in a dual way to the sets of buyers and sellers' utilities that are attainable at competitive equilibria.

In this paper we study two different cooperative solutions for this class of markets and their relationship with the set of competitive equilibrium payoffs. The two solutions differ on how a coalition of buyers and sellers can block a proposed payoff vector. Given an assignment and a coalition of buyers and sellers, some of them may be buying or selling some units of some goods to sellers or buyers outside the coalition. The notion of the Core corresponds to the notion of blocking that requires that all members of the coalition have to break all exchanges performed with all agents outside the coalition and buy or sell only with members within the coalition. In contrast, the concept of Set-wise stability corresponds to the notion of blocking that admits that members of the coalition may completely or partly keep their exchanges performed with non-members.

The Set-wise stability notion is closer to the already well established notion of stability applied to ordinal many-to-one matching models. For instance, Roth and Sotomayor [10] analyze this model and its applications to college admission problems and to labor markets for medical interns assuming that a hospital in a blocking coalition can maintain its relationships with interns outside the coalition. There is no reason to expect that the hospital, in order to make an offer to a doctor in the blocking coalition, will have to cancel the contracts it has with other doctors it has been already assigned to.

Since Set-wise blocking is easier than Core-wise blocking, the Set-wise stable set is a subset of the Core. We show here that the non-empty set of competitive equilibrium payoffs is contained in the Set-wise stable set. Hence, the Set-wise stable set as well as the Core are non-empty. Moreover, we exhibit a simple market showing that these inclusions may be strict.