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Incomplete market dynamics and cross-sectional distributions

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Abstract

The size distributions of many economic variables seem to obey the *double power law*, that is, the power law holds in both the upper and the lower tails. I explain this emergence of the double power law—which has important economic, econometric, and social implications—using a tractable dynamic stochastic general equilibrium model with heterogeneous agents subject to aggregate and idiosyncratic investment risks. I establish theoretical properties such as existence, uniqueness, and constrained efficiency of equilibrium, and provide a numerical algorithm that is guaranteed to converge. The model is widely applicable: it allows for arbitrary homothetic CRRA recursive preferences, an arbitrary Markov process governing aggregate shocks, and an arbitrary number of technologies and assets with arbitrary portfolio constraints.

JEL classification: D31; D52; D58; E21; G11

Keywords: Applied general equilibrium analysis; Emergence; Inequality; Power law; Robustness

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1. Introduction

One of the most remarkable features of the size distributions of many economic variables is that they obey the *power law*: the fraction of units exceeding size x is proportional to $x^{-\alpha}$ when x is large, where $\alpha > 0$ is the power law exponent. The power law was discovered by Pareto [51], who was studying the size distribution of income, and popularized by Gabaix [20], who provided a simple explanation of Zipf's law (power law with exponent $\alpha = 1$) for cities.[2] Recently, a new phenomenon has been discovered: the *double power law*, which means that the power law holds not only in the upper tail but also in the *lower* tail: the fraction of units *below* size x is proportional to x^{β} for some exponent $\beta > 0$ when x is small. So far, the double power law has been reported in city size [54,25], income [55,64,65], and consumption and its growth rate [67].

A question that often arises when talking about the power law is: why should we care? Here I list four reasons: (i) Such an empirical regularity is interesting in its own right and should be explained. (ii) The behavior of a system with power law distributions will be strongly influenced by the behavior of the largest units.[3] (iii) If a variable obeys the power law, its exponent determines inequality.[4] However, before we do anything about inequality (with say policy), we should understand how it is determined. Having a positive theory of the tails should come before any normative analysis. Policy coming from the wrong model may be nonsense. (iv) Since power law variables have only finitely many moments, econometric techniques that assume the existence of moments (such as GMM) might be inapplicable.[5]

What is the origin of the double power law? By introducing birth and death in a mechanistic model with geometric Brownian motion, Reed [53] showed that we can get the double Pareto distribution, whose tails satisfy the power law exactly. Benhabib, Bisin, and Zhu [8] do the same with optimizing agents in a partial equilibrium model. However, the real world is certainly more complicated than the i.i.d. world of Brownian motion. The question is, why is the double power law *robust*? This question was partly asked in the influential paper by Gabaix [20]. He argued that the power law (here in the upper tail) holds if individual units are hit by multiplicative shocks (Gibrat's law of proportionate growth [24]) and there is a small friction, such as a reflecting barrier. But he formally proved the emergence of the power law under i.i.d. assumptions, leaving the robustness issue to subsequent research. In fact, he writes "[I]t does not matter if this mean rate is time varying [..., which] is a conjecture that we firmly believe to be true. [...] However, we could not find any argument in the mathematical literature" (p. 743, footnote 13).

This paper provides an answer to the robustness question in the context of general equilibrium with incomplete markets (GEI). The logic proceeds in two steps. First, I show that in a large class of dynamic general equilibrium models with incomplete markets in which agents are hit by multiplicative aggregate and idiosyncratic shocks (*AK* models), the wealth of individual agents satisfies Gibrat's law. Second, I show that for a large class of stochastic processes, Gibrat's law and a constant probability of birth/death imply the double power law. Therefore, to the extent that

[2] See Gabaix [21] for a review of the power law.

[3] For example, Gabaix [22] shows that the idiosyncratic movements of the largest 100 firms in U.S. appear to explain a large part of the aggregate fluctuations.

[4] See Toda [65] for the connection between the power law exponents and inequality measures such as the Gini coefficient.

[5] This point is examined by Kocherlakota [33] and Toda and Walsh [67,68] in the context of the estimation of consumption-based capital asset pricing models.