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# Aggregation of preferences for skewed asset returns ☆

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**Abstract**

This paper characterizes the equilibrium demand and risk premiums in the presence of skewness risk. We extend the classical mean-variance two-fund separation theorem to a three-fund separation theorem. The additional fund is the skewness portfolio, i.e. a portfolio that gives the optimal hedge of the squared market return; it contributes to the skewness risk premium through co-variation with the squared market return and supports a stochastic discount factor that is quadratic in the market return. When the skewness portfolio does not replicate the squared market return, a tracking error appears; this tracking error contributes to risk premiums through kurtosis and pentosis risk if and only if preferences for skewness are heterogeneous. In addition to the common powers of market returns, this tracking error shows up in stochastic discount factors as priced factors that are products of the tracking error and market returns.


*JEL classification:* D52; D53; G11; G12

*Keywords:* Skewness; Portfolio; Aggregation; Stochastic discount factor; Polynomials of pricing factors; Market return

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## 1. Introduction

Empirical studies documented that securities' returns are not normally distributed; the seminal work of Harvey and Siddique [24] showed that skewness risk is an important component in the risk-premium. This renewed interest in the compensation of skewness risks and led to an active literature stream.[1] This stream typically assumes that the aggregation of preferences leads to stochastic discount factors that are polynomials in the market return, conditional on the current information set. However, it is well known that the aggregation of preferences in incomplete markets leads to a representative agent where the weight of each individual is stochastic; thus, the stochastic discount factor may actually depend on individual securities due to unspanned powers of the market return. We study in detail the aggregation of preferences with skewed returns in a single-period model of investing.

Our paper proceeds through two steps. Although the focus of our analysis is on aggregation of (heterogeneous) investors, our first step studies the case of a representative agent. We show that the common practice of using polynomials in the market return is warranted as long as the representative investor is myopic, i.e. as long as she only cares about investing over the next time period[2]; in line with this, we link our results to the beta pricing relationships proposed by Harvey and Siddique [24].

Our major second step studies equilibrium risk premiums and demand with heterogeneous investors. We derive individual demand and prove a three-fund separation theorem: agents hold the risk-free asset, the market portfolio and a new, so-called skewness portfolio, in proportions that reflect their preferences for variance and skewness risk. The skewness portfolio is the portfolio that provides the optimal hedge to the squared market return. We show that an asset's skewness risk is priced as long as the co-skewness of this asset with the market portfolio as well as the aggregate skew-tolerance do not vanish. The equilibrium contribution to the risk-premium of individual stocks is driven by their covariance with the squared market return; it supports the common practice of using a stochastic discount factor (SDF) that is quadratic in the market return.

A tracking error shows up in hedging the squared market return with the available securities; beyond average skew-tolerance, this leads to an additional lower order contribution to the risk premium through kurtosis due to the so-called cross-sectional variance of investor's skew-tolerances. Put differently, there may be an additional priced factor that contributes to skewness risk: the product of the market return with the tracking error in the squared market return. Another tracking error shows up in hedging the cubed market return with the available securities; beyond average skew-tolerance, this leads to an additional lower order contribution to the risk premium through pentosis (the fifth moment) due to the cross-sectional variance and the so-called cross-sectional skewness of investor's skew-tolerances.

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[1] For example, Dittmar [14] and Barone-Adesi et al. [3] study how skewness risk is priced; Chung et al. [10] test whether Fama–French factors proxy for skewness and higher moments; Engle [18] links the skewness that shows up in asymmetric volatility models to systemic risk; Boyer and Vorkink [6] examine the impact of skewness preference on option prices; Smetters and Zhang [42] generalize the Sharpe ratio to rank non-normal risks in portfolio evaluation.

[2] At the end of this paper (the last Subsection 5.4) we study briefly a two-period extension for the representative agent, i.e. she is no longer myopic. We argue that at least one additional priced factor shows up that is missing in polynomials of market returns: it captures intertemporal changes in the investment opportunity set. A detailed analysis of aggregation and additional priced factors in a two-period model is beyond the focus of this paper; it is carried out in Chabi-Yo et al. [8]. Further extensions to multiple periods (more than two) would be interesting as they can shed light on long-dated assets, i.e. assets with payoffs that are far in the future, see Martin [33].