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Weakly monotonic solutions for cooperative games ☆

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Abstract

The principle of weak monotonicity for cooperative games states that if a game changes so that the worth of the grand coalition and some player's marginal contribution to all coalitions increase or stay the same, then this player's payoff should not decrease. We investigate the class of values that satisfy efficiency, symmetry, and weak monotonicity. It turns out that this class coincides with the class of egalitarian Shapley values. Thus, weak monotonicity reflects the nature of the egalitarian Shapley values in the same vein as strong monotonicity reflects the nature of the Shapley value. An egalitarian Shapley value redistributes the Shapley payoffs as follows: First, the Shapley payoffs are taxed proportionally at a fixed rate. Second, the total tax revenue is distributed equally among all players.

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1. Introduction

The Shapley value (Shapley [18]) probably is the most eminent (single-valued) solution concept for cooperative games with transferable utility (TU games). Remarkably, it is not only the unique such concept that satisfies additivity, efficiency, symmetry, and the null player property, but it can be calculated as a player's average marginal contribution to coalitions. Consequently, the Shapley value satisfies a very natural monotonicity condition that conveys desirable incentive properties (Shubik [19]): whenever a player's marginal contributions weakly increase, his payoff weakly increases. Conversely, Young [21] shows that this strong monotonicity property (alongside with efficiency and symmetry) is characteristic of the Shapley value. His characterization is also remarkable since it does without additivity, which is a rather technical condition with little economic content.

Strong monotonicity implies that a player's payoff only depends on his productivity measured by marginal contributions. Hence, the Shapley value reflects individual productivity. Modern societies and institutions, however, distribute their wealth not only based on individual productivity but also on solidarity or egalitarian principles. In order to allow for such principles, strong monotonicity must be waived.

van den Brink et al. [4] reconcile monotonicity with egalitarianism. In particular, they advocate weak monotonicity as a relaxation of strong monotonicity. Weak monotonicity requires that a player's payoff weakly increases whenever his marginal contributions *and* the grand coalition's worth weakly increase. This principle is particularly attractive in view of the cooperative context and the efficiency assumption. According to efficiency, the grand coalition's worth is to be shared. If this worth does not decrease, there is no need to reduce any player's payoff. Thus, if in addition his individual productivity measured by his marginal contributions to coalitions does not decrease, there is no reason to decrease the player's payoff. van den Brink et al. [4] show that a solution satisfies efficiency, linearity (or weak covariance), anonymity, and weak monotonicity if and only if it is an egalitarian Shapley value.

The egalitarian Shapley values (Joosten [10]) are the convex mixtures of the Shapley value and the equal division value. That is, an egalitarian Shapley value redistributes the Shapley payoffs as follows: First, the Shapley payoffs are taxed proportionally at a fixed rate. Second, the total tax revenue is distributed equally among all players.

Our main result states that a value for games with more than two players satisfies efficiency, symmetry, and weak monotonicity if and only if it is an egalitarian Shapley value. Cum grano salis, this is a generalization of Young's characterization of the Shapley value. Moreover, our main result entails that linearity (respectively weak covariance) is redundant in the above characterizations of the egalitarian Shapley values by van den Brink et al. [4] if there are more than two players.

There are three other generalizations of Young's result in the literature that should be mentioned. Nowak and Radzik [15] relax the symmetry assumption and give a characterization of the weighted Shapley values (Kalai and Samet [11]). While their characterization works within the same framework as ours, de Clippel and Serrano [7] characterize a generalization of the Shapley value for coalitional games with externalities. Hart [9] provides a characterization of the Maschler–Owen consistent value for non-transferable utility games in a way that generalizes Young's theorem.

Young [21] also stresses the interest in whether weaker monotonicity criteria are met by other (non-linear) concepts as for example the nucleolus (Schmeidler [17]) or the core (Gillies [8]).