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When can expected utility handle first-order risk aversion?

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Abstract

Expected utility functions are limited to second-order (conditional) risk aversion, while non-expected utility functions can exhibit either first-order or second-order (conditional) risk aversion. We extend the concept of orders of conditional risk aversion to orders of conditional dependent risk aversion. We show that first-order conditional dependent risk aversion is consistent with the framework of the expected utility hypothesis. Our theoretical result proposes new insights into economic and financial applications such as the equity premium puzzle, the cost of business cycles, and stock market participation. Our model is compared to the rank-dependent expected utility model.

JEL classification: D81; G10; G11; G12

Keywords: Expected utility theory; First-order conditional dependent risk aversion; Background risk; Equity premium puzzle; Consumption risk in business cycles; Rank-dependent expected utility model

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1. Introduction

We present a new approach to developing an expected utility model that exhibits first-order risk aversion and explains some economic and finance puzzles. The main contribution of our model is to show that dependent background risk is sufficient to obtain a first-order risk premium distinct from a second-order premium with the standard expected utility framework. Because many economic agents are often exposed to simultaneous dependent risks (for example, an IBM employee who is considering buying IBM stock or a non-permanent employee who owns units of stock index and whose job is subject to business cycles) the sufficient condition we propose is natural for many economic agents.

The concepts of second-order and first-order risk aversion were coined by Segal and Spivak [31]. For an actuarially fair random variable $\tilde{\varepsilon}$, second-order risk aversion means that the risk premium the agent is willing to pay to avoid $k\tilde{\varepsilon}$ is proportional to k^2 as $k \to 0$. Under first-order risk aversion, the risk premium is proportional to k. Loomes and Segal [25] extend this notion to preferences about uninsured events, such as independent additive background risks. They introduce the concept of orders of conditional risk aversion. Defining $\tilde{y}$ as an independent additive risk, the conditional risk premium is the amount of money the decision maker is willing to pay to avoid $\tilde{\varepsilon}$ in the presence of $\tilde{y}$. The preference relation satisfies first-order conditional risk aversion if the risk premium the agent is willing to pay to avoid $k\tilde{\varepsilon}$ is proportional to k as $k \to 0$. It satisfies second-order conditional risk aversion if the risk premium is proportional to k^2.

First-order risk aversion implies that small risks matter. It is well known from Arrow [2] and Borch [5] that differentiable expected utility (EU) is only second order. Because expected utility theory is limited to second-order (conditional) risk aversion, it ignores many real world results. Several non-EU models that can predict first-order risk aversion behavior (i.e. rank-dependent EU and loss aversion) are being used in the economic and financial literatures to explain puzzles that EU cannot handle (see, Epstein and Zin [16], Quiggin [28], Tversky and Kahneman [34]).

For example, risk-averse individuals are reluctant to take a small actuarially favorable gamble or to invest in financial markets with a positive risk premium. These two puzzles—coward gambler (Samuelson [29]) and the stock market participation puzzle (Barberis et al. [4])—have been solved with the property of first-order risk aversion, but outside the expected utility framework. Two other significant puzzles are the equity premium puzzle (Mehra and Prescott [26]), in which asset prices are poorly explained by reasonable values of risk aversion, and the welfare cost of business cycles considered small by an expected utility representative agent who does not consider dependent background risk (Lucas [23]).

In this paper, we reinvestigate whether first-order conditional risk aversion appears in the framework of the expected utility hypothesis. The general answer to this question is positive with some weak restrictions: expected utility theory exhibits first-order risk aversion when there is a dependent background risk, but not otherwise. We extend the concepts of orders of conditional risk aversion to orders of conditional dependent risk aversion, for which $\tilde{\varepsilon}$ and the background risk $\tilde{y}$ are dependent and $\tilde{y}$ may enter the agent's utility function arbitrarily. We thus propose a new source of first-order risk aversion over general preferences. Because EU theory is simple, parsimonious, and able to explain a wide set of empirical facts, our use of first-order risk aversion is easy to interpret in many real-world applications, and the assumptions are plausible. It is now well accepted that a labor or income risk that changes over economic cycles can be interpreted as a correlated background risk for a representative agent when choosing optimal portfolio or assets in pension funds.