Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 154 (2014) 604–611
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Notes

Information and the dispersion of posterior expectations ☆

Nikolai Brandt [a], Burkhard Drees [b,c], Bernhard Eckwert [a,*], Felix Várdy [c]

[a] *Bielefeld University, Germany*
[b] *IMF, United States*
[c] *Joint Vienna Institute, Austria*

Received 29 January 2013; final version received 20 October 2014; accepted 24 October 2014

Available online 27 October 2014

Abstract

We explore the intuitive idea that more information leads to greater dispersion of posterior beliefs about the expected state of the world. First, we show that two dispersion orders that have been widely used as informativeness criteria do not satisfy the desirable property of ordinality of states (OS), i.e., invariance to increasing monotone state transformations. Then, for the class of monotone information systems, we characterize the weakest information criteria that respect OS and imply the dispersion orders. Our characterizations consist of intuitive conditions on the joint distributions of signals and states. Because of OS, the information criteria induce the dispersion orders not only on the posterior expectations of states, but also of state *utilities*, under any strictly increasing vNM utility function.

JEL classification: D80; D81

Keywords: Information; Dispersion orderings; Decision making under uncertainty

☆ We would like to thank seminar participants at Bielefeld, Konstanz, and the Technical University of Chemnitz. We especially wish to acknowledge receiving helpful suggestions from the editor, Alessandro Pavan, and two anonymous referees.

* Corresponding author at: Dept. of Economics, Bielefeld University, Universitätsstr. 25, 33615 Bielefeld, Germany.
E-mail address: beckwert@wiwi.uni-bielefeld.de (B. Eckwert).

http://dx.doi.org/10.1016/j.jet.2014.10.004
0022-0531/

1. Introduction

In Bayesian decision problems, informativeness and the dispersion of posterior beliefs appear to be linked. Intuitively, the more informative is an information system, the more 'aggressive' is Bayesian updating and, hence, the more dispersed tend to be the posterior expectations. Therefore, some recent papers *equate* informativeness with the dispersion of posterior expectations. That is, they use dispersion orderings as information criteria (e.g., Baker [1]; Eckwert and Zilcha [4]; Ganuza and Penalva [5]; Li [10]).

In this paper, we show that the resulting information criteria are, in general, not invariant to increasing state transformations, even though these transformations fully preserve the information that is revealed. Other well-known information criteria, such as sufficiency (Blackwell [3]), effectiveness/accuracy (Lehmann [9]; Persico [13]), and efficiency (Kim [8]), do satisfy this desirable invariance property, which we call *ordinality of states* (OS).

To remedy this drawback of equating information with dispersion, for the class of monotone information systems, we characterize the weakest information criteria that respect OS *and* imply the two dispersion orders studied in Ganuza and Penalva [5]. These characterizations consist of intuitive conditions on the joint distributions of signals and states. The information criterion that characterizes mean-preserving spread (MPS) dispersion and OS we call the 'weak' informativeness criterion. The information criterion that characterizes supermodular (SM) dispersion and OS we call the 'strong' informativeness criterion. Because of OS, a weakly (strongly) more informative system induces greater MPS (SM) dispersion not only on the posterior expectations of states, but also of state *utilities*, under any strictly increasing vNM utility function.

Finally, we show that an information system consisting of a convex combination of systems that are informationally ordered has intermediate informativeness and dispersion. In this sense, the ray between ordered systems is ordered.

2. Model and dispersion orders

Let $\tilde{q}$ be a random variable with realization q, describing the state of the world. The state variable $\tilde{q}$ takes values in $Q := [\underline{q}, \bar{q}] \subset \mathbb{R}$. Denote by $h(q) > 0$ the pdf of the prior of $\tilde{q}$ and by $H(q)$ its cdf. At decision time, the realized state is unknown. However, a real-valued, noisy signal $\tilde{s}$ that is correlated with the state is observed.

An *information system* is a joint density $f(s, q)$ of signals and states with associated cumulative distribution function $F(s, q) = \Pr[\tilde{s} \le s, \tilde{q} \le q]$. Denote by $f(s|q)$ and $F(s|q)$ the conditional signal density and distribution functions, respectively. The marginal distribution of the signal has density $f(s) = \int_Q f(s, q)\, \mathrm{d}q$. Without loss of generality, we assume that no signal is redundant, i.e., $f(s) > 0\ \forall s$. Similarly, we can freely assume that signals are uniformly distributed on $S = [0, 1]$, i.e., $f(s) = 1$ for all s.[1] Then, by Bayes' rule, the posterior pdf of $\tilde{q}$ is $h(q|s) = f(s, q)$, while the posterior cdf is $H(q|s) := \int_{\underline{q}}^{q} h(q'|s)\, \mathrm{d}q'$.

[1] Indeed, any information system with strictly positive marginal signal density can be transformed into an equivalent system with uniformly distributed signals on [0, 1]. For a given prior and information system $f : S \times Q \to \mathbb{R}_+$, consider the transformation $\tilde{\pi} := F \circ \tilde{s}$, where F is the cdf of the marginal signal density. Under our assumptions, F is strictly increasing and continuous. Therefore, $\tilde{\pi}$ is uniformly distributed on [0, 1], and signal monotonicity is preserved since the transformation is monotonic. Moreover, the transformed signal $\tilde{\pi} = F(\tilde{s})$ fully reveals the realizations of the original signal, $\tilde{s}$, in terms of quantiles and, therefore, conveys exactly the same information. Lehmann [9] shows that even if F is not continuous or strictly increasing, it is always possible to define an informationally equivalent system such that the corresponding marginal signal distribution *is* continuous and strictly increasing.