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The dynamic instability of dispersed price equilibria ☆

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Abstract

We adopt an evolutionary framework to explain price dispersion as a time varying phenomenon. By developing a finite strategy analogue of the Burdett and Judd (1983) price dispersion model, we show that all dispersed price equilibria are unstable under the class of perturbed best response dynamics. Instead, numerical simulations using the logit dynamic show that price dispersion manifests itself as a limit cycle. We verify that limit cycles persist even when the finite strategy model approaches the original continuous strategy model. For a particularly simple case of the model, we prove the existence of a limit cycle.

JEL classification: C72; C73; L11

Keywords: Price dispersion; Evolutionary game theory; Logit dynamic

1. Introduction

In this paper, we use evolutionary game theory to explain price dispersion in the Burdett and Judd [11] model as a limit cycle. Price dispersion, under which different sellers charge different prices for the same homogeneous good is a commonly observed phenomenon. Hopkins [29] provides detailed evidence of the prevalence of price dispersion, including its persistence among firms that sell through the Internet (Baylis and Perloff [4] and Baye et al. [3]). Price dispersion is very puzzling since it seemingly contradicts the "law of one price" of elementary microeco-

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nomics. Various models explain price dispersion as an equilibrium phenomenon. The common feature of these models is the presence of heterogeneity among consumers, with some consumers willing to pay a price that is not necessarily the lowest prevailing.[1] Instead of undercutting each other in Bertrand like competition, sellers can earn more by sticking to a higher price and selling to the fraction of consumers who would be willing to buy at that price. We call the resulting mixed equilibrium a dispersed price equilibrium.

There are however reasons to be circumspect about an explanation that relies on the notion of a mixed equilibrium. For this explanation to be plausible, it must be possible for a large, technically uncountably infinite, population of sellers to be able to coordinate on a probability distribution over an immense number of prices. This is clearly a coordination problem of a much greater order of complication than coordinating on a pure equilibrium. Nor is it obvious how sellers randomize over such a large number of alternative strategies.

It is therefore necessary to impose a sterner test on the mixed equilibrium prediction before accepting its validity. One way to test it is to postulate an initial scenario in which the population distribution over the various prices (the social state) is close to but not exactly the same as a mixed equilibrium; an overwhelmingly more likely event than exact coordination on the equilibrium. We then introduce a dynamic process with appropriate micro foundations that allows us to judge whether the social state moves towards or diverges away significantly from the mixed equilibrium. If indeed we observe the latter outcome, then we need to fashion a new explanation for observed price dispersion. This is the approach of evolutionary game theory in which individual agents, as members of large populations, periodically revise their strategies using simple revision protocols. This approach, by not requiring large populations of agents to possess the knowledge or ability to coordinate on a precise equilibrium, is more respectful of the bounded nature of human rationality. The revision protocol generates an evolutionary dynamic which is an ODE system in the social state with the set of Nash equilibria as rest points.

Indeed, there is a precedent to adopting this approach in the study of price dispersion by Hopkins and Seymour [30] who analyze the Burdett and Judd [11] and the Varian [41] models using the class of Positive Definite Adaptive (PDA) dynamics of which the replicator dynamic (Taylor and Jonker [40]) is the prototypical representative. Such dynamics are generated by agents imitating others using more successful strategies. Their analysis reveals that the mixed equilibria of these models are unstable under these dynamics; i.e. the social state starting near any of these equilibria diverges away. They explain this result using the notion of positive definite games; i.e. games in which near a mixed equilibrium, any mutant strain that plays the Nash equilibrium finds itself at a payoff disadvantage. This is of course the reverse of Maynard Smith's notion of Evolutionary Stable State. Evolutionary dynamics which respect such payoff differentials will therefore take the population away from a mixed equilibrium.

To fully understand price dispersion, one needs to go beyond the negative conclusion of Hopkins and Seymour [30] that mixed equilibria do not suffice as an explanation of this phenomenon. Recent empirical (Lach [32]) and experimental (Cason et al. [13]) work suggest that price dispersion is a cyclical phenomenon. They find that the proportion of sellers charging any price keeps oscillating as a regular cycle. Such a phenomenon is of course readily understood as a limit cycle in evolutionary game theory. The *primary* contribution of this paper is that we are

[1] Some of these models are Salop and Stiglitz [36], Varian [41], Burdett and Judd [11], Rob [35], Stahl [39], Wilde [43], Bénabou [7]. As examples of consumer heterogeneity, we can cite the number of prices consumers sample before purchasing (Burdett and Judd [11]), or search cost (Salop and Stiglitz [36], Stahl [39]). Hopkins [29] provides a survey of the theoretical literature on price dispersion.