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Large deviations and multinomial probit choice ☆

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Abstract

We consider a discrete choice model in which the payoffs to each of an agent's n actions are subjected to the average of m i.i.d. shocks, and use tools from large deviations theory to characterize the rate of decay of the probability of choosing a given suboptimal action as m approaches infinity. Our model includes the multinomial probit model of Myatt and Wallace (2003) [5] as a special case. We show that their formula describing the rates of decay of choice probabilities is incorrect, provide the correct formula, and use our large deviations analysis to provide intuition for the difference between the two.

JEL classification: C25; C73

Keywords: Discrete choice theory; Large deviations theory; Multinomial probit model; Stochastic evolutionary game theory; Stochastic stability

1. Introduction

In a paper in this journal, Myatt and Wallace [5] consider a model of stochastic evolution based on the multinomial probit model. Agents in their model optimize after their payoffs are subjected to i.i.d. normal shocks, and their analysis focuses on the agents' long run behavior as

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the variance of the shocks is taken to zero. Compared to other models of choice used in stochastic evolutionary game theory, the multinomial probit model introduces a novel feature: the rate of decay in the probability of choosing a suboptimal strategy is neither independent of payoffs, as in the mutation models of Kandori, Mailath, and Rob [3] and Young [7], nor dependent only on the gap between its payoff and the optimal strategy's payoff, as in the logit model of Blume [1], but can depend on the gaps between its payoff and those of all better performing strategies.[1] The foundation of the analysis in Myatt and Wallace [5] (henceforth MW) is their Proposition 1, which characterizes the rates of decay of multinomial probit choice probabilities as the shock variance approaches zero. Their characterization is based on a direct evaluation of the limit of the relevant multiple integral.

In this note, we introduce a model of choice in which the payoffs to each of an agent's n actions are subject to the average of m i.i.d. shocks. One can interpret this average as representing the net effect of many small payoff disturbances. Our model comes equipped with a natural parameterization of the small noise limit: as the number of shocks grows large, the probability of a suboptimal choice approaches zero. Using techniques from large deviations theory, we derive basic monotonicity and convexity properties of the rates of decay of choice probabilities, and we obtain a simple characterization of the rates themselves.

Since the average of independent normal random variables is itself normally distributed, MW's model of choice can be obtained as a special case of ours. Our analysis reveals that MW's formula for the rate of decay of multinomial probit choice probabilities is incorrect. We derive the correct formula for the rate of decay, and we offer an intuitive explanation for the difference between the formulas using the language of large deviations theory.

2. Analysis

2.1. Large deviations and Cramér's theorem

Let $\{Z^l\}_{l=1}^{\infty}$ be an i.i.d. sequence of random vectors taking values in $\mathbb{R}^n$. Each random vector Z^l is continuous with convex support, with a moment generating function that exists in a neighborhood of the origin.

Let $\bar{Z}^m = \frac{1}{m}\sum_{l=1}^{m} Z^l$ denote the mth sample mean of the sequence $\{Z^l\}_{l=1}^{\infty}$. The weak law of large numbers tells us that $\bar{Z}^m$ converges in probability to its mean vector $\mu \equiv \mathbb{E}Z^l \in \mathbb{R}^n$. We now explain how methods from large deviations theory can be used to describe the rate of decay of the probability that $\bar{Z}^m$ lies in a given set $U \subset \mathbb{R}^n$ not containing μ.

The *Cramér transform* of Z^l, denoted $R : \mathbb{R}^n \to [0, \infty]$, is defined by

$$R(z) = \sup_{\lambda \in \mathbb{R}^n} \big(\lambda' z - \Lambda(\lambda)\big), \quad \text{where } \Lambda(\lambda) = \log \mathbb{E} \exp\big(\lambda' Z^l\big).$$

Put differently, R is the convex conjugate of the logarithmic moment generating function of Z^l. It can be shown that R is a convex, lower semicontinuous, nonnegative function that satisfies $R(\mu) = 0$. Moreover, R is finite, strictly convex, and continuously differentiable on the interior of the support of Z^l, and is infinite outside the support of Z^l.[2]

[1] Related models also appear in unpublished work of Ui [6].

[2] These properties of the Cramér transform and Cramér's theorem can be found in Section 2.2 of Dembo and Zeitouni [2]. In particular, the finiteness, strict convexity, and smoothness of R on the interior of its domain follow from the assumptions that Z^l has convex support and that its moment generating function exists—see Exercises 2.2.24 and 2.2.39 in [2].