Available online at www.sciencedirect.com

ScienceDirect

Journal of Economic Theory 159 (2015) 137–162

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Strategy-proof assignment of multiple resources

Albin Erlanson [a,*], Karol Flores-Szwagrzak [b]

[a] *Bonn University, Germany*
[b] *University of Southern Denmark, Denmark*

Received 31 March 2014; final version received 7 May 2015; accepted 25 May 2015

Available online 1 June 2015

Abstract

We examine the strategy-proof allocation of multiple resources; an application is the assignment of packages of tasks, workloads, and compensations among the members of an organization. In the domain of multidimensional single-peaked preferences, we find that any allocation mechanism obtained by maximizing a separably concave function over a polyhedral extension of the set of Pareto-efficient allocations is strategy-proof. Moreover, these are the only strategy-proof, unanimous, consistent, and resource-monotonic mechanisms. These mechanisms generalize the parametric rationing mechanisms (Young, 1987), some of which date back to the Babylonian Talmud.

JEL classification: D70; D63; D47; D61

Keywords: Strategy-proofness; Consistency; Resource-monotonicity

1. Introduction

This paper introduces incentive compatible mechanisms to allocate multiple resources. Applications include the assignment of bundles of tasks, workloads, support personnel, and compensations among a research staff or among an academic department's faculty. In these allocation problems cash transfers are constrained or impossible, resources are not necessarily disposable,

* Corresponding author.
E-mail addresses: aerlanso@uni-bonn.de (A. Erlanson), karolszw@sam.sdu.dk (K. Flores-Szwagrzak).

http://dx.doi.org/10.1016/j.jet.2015.05.016
0022-0531/

and preferences cannot be assumed to be monotone. This paper studies the case where preferences over assignments are "multidimensional single-peaked": an agent has an ideal amount of each resource; increases in the amount of a single resource leaving her below the ideal for that resource make her better off, increases beyond it make her worse off.

As in most economic design problems, the relevant information to evaluate the welfare impact of choosing a mechanism, the preferences of the agents involved, is privately held. Successful real-life mechanisms overcome this difficulty and the resulting incentives for manipulation by making truthful preference revelation a dominant strategy. These mechanisms are known as *strategy-proof* and examples include the matching mechanisms in school choice (Abdulkadiroğlu and Sönmez, 2003; Pathak and Sönmez, 2008; Abdulkadiroğlu et al., 2009), kidney exchange (Roth et al., 2004, 2005), and entry level labor markets (as surveyed by Roth, 2002). The focus on dominant strategy incentive compatibility is due to its minimal assumptions about agents' knowledge and behavior. Since reporting preferences truthfully is a dominant strategy, equilibrium behavior does not depend on beliefs, common knowledge of rationality and the information structure, etc. This gives a predictive power and a robustness that are important for practical mechanism design (Wilson, 1987; Bergemann and Morris, 2005).

Unfortunately, in the resource allocation problems studied here, sequential dictatorship is essentially the only strategy-proof and efficient mechanism.[1] This mechanism is neither individually rational nor equitable. Often these distributional objectives will override efficiency and thus exclude this mechanism. In other words, the mechanism designer faces a tradeoff between efficiency and any other objective she may want to implement. This paper describes the class of strategy-proof mechanisms that avoid a number of drawbacks once efficiency is relaxed.

First, we exclude the most inefficient mechanisms. Every mechanism in the class is *unanimous*: if an allocation yielding each agent her ideal assignment is feasible, then the mechanism delivers this allocation. Though sequential dictatorship is the only efficient mechanism in the class, strongly egalitarian mechanisms are also members.

Second, we exclude mechanisms that recommend allocations contradicting each other. A mechanism is *consistent* if its recommendations in problems involving different groups of agents and resources are coherent.[2]

Third, we exclude mechanisms not responding well to changes in the availability of resources. A mechanism is *resource-monotonic* if all agents are made at least as well off in response to certain changes in the availability of resources that can unambiguously make everyone better off. This embodies a basic solidarity notion.[3]

Every strategy-proof, unanimous, consistent, and resource-monotonic mechanism is specified by a list of strictly concave functions (Theorem 1). These functions determine how heavily an agent's welfare is weighed against another's. According to the scarcity of resources, a function is drawn from this list for each agent and each resource. The sum of these functions is then maximized subject to efficiency constraints. The unique maximizer is the allocation recommended by

[1] Sequential dictatorship is the mechanism whereby agents are arranged sequentially, and resources are allocated accordingly. The first agent in the sequence is assigned her best possible bundle. Conditional on this, the second agent is assigned her best possible bundle, and so forth.

[2] Consistency is one of the most thoroughly studied principles in resource allocation. See Thomson (2011a) for an overview. Balinski (2005) and Thomson (2012) discuss the normative content of consistency which Balinski calls "coherence".

[3] See Thomson (2011b) for an overview of solidarity properties in economic environments.