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A dynamic generalization of Becker's assortative matching result ☆

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Abstract

This paper considers a dynamic matching model in which each agent's future productivity depends in part on their current match, as in labor markets, schooling, intergenerational marriage markets, and other environments. The Planner's endogenous rankings of human distributions are characterized. These Planner rankings are then used to develop sufficient conditions for positive assortative matching to be dynamically efficient. One lesson that emerges is that complementarity assumptions alone are insufficient for a robust sorting theory — the curvature of the static production function is also critical to determine optimal sorting patterns. In addition, the Planner's ranking of distributions over human capital yield characterizations of individual attitudes toward human capital gambles in an associated market equilibrium. Finally, the implied dynamics for (1) individual wages and (2) wage distributions across age cohorts are characterized.

JEL classification: C61; C78; D13; D31

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1. Introduction

In 1973, Becker introduced a pairwise matching model with type-dependent perfectly divisible match output. Becker showed that if productive types are complements, then equilibrium

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sorting is positive: the highest types match together, then the next highest, etc. In the decades since, Becker's matching model has been embellished in many ways and fruitfully applied to study sorting in marriage markets, job markets, schools, and neighborhoods. His original theoretical prediction has proven to be quite robust: given sufficient complementarity assumptions on the match production function, positive sorting obtains across a wide spectrum of related matching models.[1]

Anderson and Smith (2010) consider an infinite horizon matching model with *peer effects*, allowing the evolution of individual agent characteristics to depend on their match partners. Intuitively, workers may learn how to produce more efficiently from star coworkers or pick up bad habits from less productive partners. Anderson and Smith (2010) establish existence and the welfare theorems, but they have little to say about sorting patterns. In fact, their main matching result is that sorting cannot obtain with sufficiently patient agents in an incomplete information special case of their general model. In contrast, one contribution of the current paper is characterizing when perfect sorting obtains with dynamic matching and peer effects.

In static matching models, the match value function is exogenous. Here the value to any match depends on both the static production function and continuation values. These continuation values depends on the human capital transition function and the equilibrium value of human capital. Thus, the equilibrium value of human capital must first be characterized before analyzing sorting incentives. Given the welfare theorems, we characterize values using the Planner's problem. Specifically, the Planner chooses the distribution over matches in each period to maximize the present discounted value of aggregate output, taking the current distribution over human capital as given. In doing so, the Planner may trade off lower static production in the current period for a *better* distribution over human capital in the future. But what is a better distribution over human capital? Lemma 1 provides an answer. An immediate implication of the scaling of productive types (higher is better) is that first order shifts in the human capital distribution increase discounted aggregate output (Lemma 1: (a)). The Planner's ranking of second order changes in the distribution of human capital is more nuanced and depends on both the curvature of the production function and stochastic transitions in human capital (Lemma 1: (b)). For example, the lemma offers conditions under which mean-preserving spreads in the distribution over human capital reduce total discounted output: there need not be a conflict between efficiency and equity when it comes to human capital distributions.[2]

Having characterized values, we turn to characterizations of sorting. In static matching models, sorting follows from complementarity (supermodularity) of the production function. In the dynamic model, we must consider both static and dynamic complementarity. Theorem 1 provides the transition complementarity assumption sufficient to imply positive sorting for any static production function that is both increasing and supermodular. However, the subsequent discussion underscores that the required complementarity condition on the transition kernel is extremely strong, unlikely to be satisfied in most economic environments. Recognizing this, Theorem 2

[1] The definition of sorting is context-dependent. Chade (2005) provides a definition suitable for a noisy environment. See Legros and Newman (2002) for several alternative notions of positive sorting.

[2] There is a small literature on efficient macroeconomic inequality. In Atkeson and Lucas (1992) inequality makes private information revelation incentive compatible. Welch (1999) is mostly an empirical piece that draws a distinction between unequal outcomes and unequal opportunity, arguing that unequal outcomes can provide incentives for effort, education, etc. Eeckhout (2006) considers a matching framework with an observable but inessential parameter. Equilibria in which sorting is conditioned on this inessential parameter can payoff dominate equilibria in which sorting is blind to the parameter. Again, inequality is efficient.