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Iterated generalized half-dominance and global game selection ☆

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Abstract

We offer an equilibrium characterization of a general class of global games with strategic complementarities. The analysis highlights a form of acyclicity in the interim belief structure of global games, which allows us to formalize a selection criterion, *iterated generalized half-dominance*. This criterion is shown to be a unique global game selection when noise distributions satisfy a regularity condition. A similar logic also applies to the perfect foresight dynamics of Matsui and Matsuyama (1995); an iterated generalized half-dominant equilibrium is a unique globally stable state when agents are patient enough. The criterion is especially useful for games with more than two asymmetric players, and can be easily applied to local interaction games with an arbitrary network structure.

JEL classification: C72; C73; D82

Keywords: Equilibrium selection; Global games; Perfect foresight dynamics; Networks; Asymmetric games

1. Introduction

Many games admit multiple Nash equilibria, and there is now a vast body of literature that aims to select a unique equilibrium from them. The theory of global games is one of the leading

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approaches and has been used in various applications.[1] The global game approach extends a complete information game by allowing payoffs to depend on an unobservable state, where each player privately observes a noisy signal about the state. Carlsson and van Damme (1993) study 2×2 games and show that as the noise level parameter goes to zero, the set of rationalizable actions in a global game shrinks to the risk-dominant action in the complete information game at each state. The key mechanism behind equilibrium selection is driven by the structure of interim beliefs held by "threshold types," who are indifferent between two actions upon receiving a signal. Under symmetric payoffs and binary actions, a threshold type has a Laplacian belief, whereby opponents' actions are uniformly distributed (Morris and Shin, 2003). This property leads to a tractable characterization of equilibrium selection that is easily applicable to various applications.

In this paper we allow for many asymmetric players and actions. While this generalization is clearly important for applications, its characterization is not fully developed in the literature, because one has to deal with more complicated belief structures that involve multiple threshold types. Our analysis uncovers a general logic underlying global game selection by highlighting a form of acyclicity in pairwise comparison of players' interim beliefs. This inspires us to propose a equilibrium selection criterion of *generalized half-dominance (GH-dominance)*. It requires each player's action to be strictly optimal if she believes that each marginal probability of her opponent choosing the equilibrium action is more than $\frac{1}{2}$.[2] As opposed to the **p**-dominance criterion that we will discuss shortly, this criterion is only concerned with beliefs about each opponent's action separately and does not involve beliefs about opponents' collective action profiles. To illustrate the intuition behind why this reasoning about each opponent's separate action is relevant in global game selection, we consider the following example. It shows that, when a GH-dominant equilibrium exists, players cannot coordinate on multiple equilibrium actions using payoff-irreverent private signals similar to a global-game type information structure.

Example 1. Fix a complete information game, and suppose that there is a GH-dominant equilibrium, where a_i^* denotes the equilibrium action of player i. We endow this game with a payoff-irrelevant state variable θ that follows the (improper) uniform prior over $\mathbb{R}$; Each player i privately observes signal $t_i = \theta + \eta_i$, where each η_i is i.i.d noise with a positive density. Consider threshold strategies in which each player i chooses a_i^* if $t_i < \bar{t}_i$, and another action if $t_i > \bar{t}_i$. We argue that such a strategy profile cannot be an equilibrium. Note that in such an equilibrium, i should be indifferent between choosing a_i^* and another action at signal $t_i = \bar{t}_i$. Take an agent i^* with $\bar{t}_{i^*} = \min_i \bar{t}_i$. (See Fig. 1.) If i^* observes signal $t_{i^*} = \bar{t}_{i^*}$, she believes that for any $i \neq i^*$, $t_i < \bar{t}_i$ holds with probability greater than $\frac{1}{2}$. That is, she believes that each opponent i's marginal probability of choosing a_i^* is more than $\frac{1}{2}$. Since $a_{i^*}^*$ is a GH-dominant action, she has a strict incentive to choose it at signal $\bar{t}_{i^*}$, which is a contradiction. □

The example illustrates how the "contagion" argument forces GH-dominant actions to be played at all interim types of players if these actions are known to be chosen at low enough signals. A key feature in this example is the existence of an extremal threshold type $\bar{t}_{i^*}$ who believes each opponent's signal t_i to be less than its threshold $\bar{t}_i$ with marginal probability more than $\frac{1}{2}$. As we will see, the conclusion carries over beyond i.i.d noises, as long as we impose a

[1] See Morris and Shin (2003) for a survey. Some recent papers incorporate endogenous information, which can prevent unique selection (e.g., Angeletos and Pavan, 2013).

[2] This generalizes the notion of half-dominance (Morris et al., 1995), which is defined for symmetric two-player games.