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A theoretical foundation for the undercut-proof equilibrium ☆

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Abstract

This paper develops a theoretical foundation for the undercut-proof equilibrium (see Shy, 1996, 2002; Morgan and Shy, 2015). In a general spatial setting, the set of undercut-proof prices is equivalent to the *core* of a non-transferable utility coalitional-game, played on the set of outcomes that are feasible in Bertrand competition. The result depends critically on two conditions: First, firms must have unlimited capacity and constant marginal costs. Second, the goods produced by firms must only be differentiated by the spatial characteristics of the market. An application to network markets shows how the undercut-proof equilibrium can be used to describe stable price dispersion and persistent performance differences.

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1. Introduction

Price formation in oligopoly markets is typically analysed through the lens of Bertrand competition. Arguably the most appealing feature of a Bertrand–Nash equilibrium in pure strategies, is ex-post stability. Once all prices are revealed to the market, no firm has an incentive to unilater-

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ally alter its price. Ex-post stability provides an intuitive rationale for using a one-shot Bertrand price-setting game to analyse prices that persist over a relatively long time-horizon.

There are, however, many models of competition for which pure-strategy Bertrand–Nash equilibria cannot be found. Prominent examples include the switching-cost model (Shilony, 1977), Hotelling competition with linear transport costs and closely spaced firm (d'Aspremont et al., 1979), and monopolistic competition with captive consumers (Varian, 1980). While Bertrand–Nash equilibria in mixed strategies do exist for these models, the resulting prices are not ex-post stable. Upon observing the prices that emerge from a mixed-strategy equilibrium, it must be the case that at least one firm would gain by unilaterally altering its price.

Several authors have identified the need for an alternative equilibrium concept; one that is consistent with the Bertrand–Nash equilibrium where it exists in pure strategies, but that is capable of identifying ex-post stable prices elsewhere. Shy (1996, Ch. 7) (see also Shy, 2002; Morgan and Shy, 2015) argues that prices should be considered ex-post stable if no firm can profitably undercut its rivals. Shy calls the firm preferred undercut-proof prices, the *undercut-proof equilibrium*. Similar equilibrium concepts have been proposed by Davis et al. (2004) and Iskakov and Iskakov (2012).

While the undercut-proof equilibrium is certainly intuitively appealing, hitherto it has lacked a robust theoretical foundation. A natural critique of the undercut-proof equilibrium, and similar equilibrium concepts, is that they are motivated by assumptions on timing and choice sets, that are not then incorporated into the formal model.[1] Consequently, it is not readily apparent under what conditions the equilibrium concept is valid.

The purpose of this paper is to provide a theoretical foundation for the undercut-proof equilibrium. The approach adopted here is to take, as given, the set of outcomes that are feasible in Bertrand competition. No additional restrictions are placed on the choices available to firms and consumers. Neither are payoff functions altered in any way. The only difference with previous approaches is the concept of ex-post stability employed. Specifically, prices are regarded as ex-post stable if no subset of market participants can improve their respective payoffs by leaving the market, and trading amongst themselves.

This definition of ex-post stability describes the *core* of a non-transferable utility (NTU) coalitional-game, played on the set of outcomes that are feasible in Bertrand competition. Hamilton et al. (1991) used the core in this way to determine optimal location choice on the Hotelling line with anonymous consumer locations.[2] Similar techniques have been used elsewhere in the literature to identify stable tax rates (Guesnerie and Oddou, 1981), and natural monopoly prices (Spulber, 1986).

The primary contribution of this paper is to show that, for a broad class of spatial model, the set of prices that give rise to core outcomes is exactly the set of undercut-proof prices. This result supports the use of the undercut-proof equilibrium in switching-costs models with arbitrary numbers of firms and consumers. Moreover, it extends to many familiar models of spatial competition, as well as competition in network markets.

The core equivalence result depends critically on two conditions: First, firms must have unlimited capacity and constant marginal costs. This means that consumers never come into competition with one and other for scarce output; a scenario which would create upward pres-

[1] For example, both Shy (1996, Ch. 7) and Davis et al. (2004) argue that, once announced, prices are upward sticky. While Iskakov and Iskakov (2012) suggest that a firm's primary concern is to prevent being pressed out of the market.

[2] Hamilton et al. (1991) (see also Stuart, 2004) argue that this approach is appropriate when consumers are anonymous, constraining each firm to offer the same price to all consumers.