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On the Phelps–Koopmans theorem ☆

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Dedicated to the memory of David Cass: mentor, friend and an extraordinary economic theorist

Abstract

We examine whether the Phelps–Koopmans theorem is valid in models with nonconvex production technologies. We argue that a nonstationary path that converges to a capital stock above the smallest golden rule may indeed be efficient. This finding has the important implication that "capital overaccumulation" need not always imply inefficiency. Under mild regularity and smoothness assumptions, we provide an almost-complete characterization of situations in which every path with limit in excess of the smallest golden rule must be inefficient, so that a version of the Phelps–Koopmans theorem can be recovered. Finally, we establish that a nonconvergent path with limiting capital stocks above (and bounded away from) the smallest golden rule can be efficient, even if the model admits a unique golden rule. Thus the Phelps–Koopmans theorem in its general form fails to be valid, and we argue that this failure is robust across nonconvex models of growth.

JEL classification: C61; O41

Keywords: Capital overaccumulation; Inefficiency; Phelps–Koopmans theorem; Nonconvex production set

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1. Introduction

The phenomenon of inefficiency of intertemporal consumption streams has been traditionally identified with the overaccumulation of capital. In fact, this message is strongly conveyed in two famous papers on efficiency by Malinvaud [4] and Cass [2].[1]

In the standard aggregative model of economic growth, the Phelps–Koopmans theorem provides one of the most well-known sufficient conditions for inefficiency.[2] This result was conjectured by Phelps [6], and its proof, based on a proof provided by Koopmans, appeared in Phelps [7]. It states that if the capital stock of a path is above, and bounded away from, the golden rule stock, from a certain time onward, then the path is inefficient.[3]

The purpose of this paper is to examine the validity of the Phelps–Koopmans theorem in aggregative models which allow for nonconvexity of the production set.[4] Of course, nonconvexity is no impediment to the existence of a golden rule provided that suitable end-point conditions hold (which we shall assume). Indeed, there may be several; we will refer to the smallest of them as the *minimal* golden rule. The Phelps–Koopmans theorem can then be restated in three progressively stronger formats:

I. Every stationary path with capital stock in excess of the minimal golden rule is inefficient.
II. A path is inefficient if it converges to a limit capital stock in excess of the minimal golden rule.
III. A path is inefficient if it is lies above (and bounded away from) the minimal golden rule from a certain time onwards.

Obviously, version III nests II, which in turn nests version I.

It is very easy to see that the weakest version I of the Phelps–Koopmans theorem must be true. But the analysis in Section 3.1 shows that version II of the theorem is generally false. We present there an example of an *efficient* path that converges to a limit stock that exceeds the minimal golden rule. This has the important implication that the phenomenon of "overaccumulation of capital" need not always imply inefficiency.

Since this finding might appear somewhat surprising, we try to convey an intuition for the result. Consider a setting with multiple golden rule stocks, and construct a path whose capital stock converges to some *nonminimal* (and therefore, by version I, inefficient) golden rule stock from above in such a way that at each period, the consumption level on the path in every period exceeds golden rule consumption.[5] If the path were inefficient, then there would be a path starting from the same initial stock, which dominates it in terms of consumption (in the efficiency ordering). This forces the capital stock of the dominating path to go below (and stay below) the

[1] In fact, one might make a case that this message can be inferred from the titles of the two papers.

[2] In awarding the Prize in Economic Sciences in Memory of Alfred Nobel for 2006 to Edmund Phelps, the Royal Swedish Academy of Sciences referred to this result as follows: "Phelps ... showed that all generations may, under certain conditions, gain from changes in the savings rate."

[3] The expression "overaccumulation of capital" in this literature refers therefore to accumulation of capital in excess of the golden rule capital stock in this precise sense. Thus, any stationary path with capital stock in excess of the golden rule capital stock, overaccumulates capital and is inefficient. The Phelps–Koopmans theorem generalizes this result to nonstationary paths.

[4] See Mitra and Ray [5] for a description of the setting, which does not assume smoothness of the production function, and does not place restrictions on the types of nonconcavities allowed.

[5] The consumption levels must, of course, converge to the golden rule consumption level over time.