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The difference indifference makes in strategy-proof allocation of objects ☆

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Abstract

We study problems of allocating objects among people. Some objects may be initially owned and the rest are unowned. Each person needs exactly one object and initially owns at most one object. We drop the common assumption of strict preferences. Without this assumption, it suffices to study problems where each person initially owns an object and every object is owned. For such problems, when preferences are strict, the "top trading cycles" algorithm provides the only rule that is efficient, strategy-proof, and individually rational Ma (1994) [1]. Our contribution is to generalize this algorithm to accommodate indifference without compromising on efficiency and incentives.

JEL classification: C71; C78; D71; D78

Keywords: Strategy-proofness; Indivisible goods; Indifference; Housing market; House allocation; Existing tenants; Top trading cycles

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1. Introduction

We study problems of allocating objects to people. In particular, we are interested in problems where each person needs a single object and initially owns at most one object. Furthermore, these problems do not involve other goods (divisible or indivisible) that can be allocated along with the objects. Think of the allocation of seminar slots, on-campus apartments, or organs for transplant. This is a broad class of problems including, at the extremes, those where no one initially owns an object [2] and those where everyone initially owns an object [3]. We account for people being indifferent between objects. Consequently, as we discuss in Section 3, it turns out that it suffices to consider problems where everyone initially owns an object and every object is owned.

Most of the related literature deals only with problems where no one is ever indifferent. We argue that there are many real-world situations where preferences do exhibit indifference. For instance, if preferences are based on coarse descriptions (say, from a housing brochure), there may be insufficient information to break ties. Alternatively, if preferences are based on checklists of criteria (like blood and tissue types for organ transplant), distinct objects satisfying exactly the same criteria are equivalent. Appropriate design of rules should take indifferences into account since breaking ties arbitrarily may lead to inefficiencies.

Our contribution is to define a class of rules that are *strategy-proof, Pareto-efficient*, and *individually rational* for such problems. Our rules are based on a new algorithm. Alcalde-Unzu and Molis [4] have simultaneously and independently proposed another such algorithm. The compatibility of these three properties had been an open question until these works. In fact, an earlier result [5] is that these three properties are incompatible with *non-bossiness*.

Our algorithm is a novel adaptation of Gale's "top trading cycles" algorithm [3]. The *top trading cycles* algorithm, which is defined only when there are no indifferences, proceeds by iterating the following: Each person "points" at the person who owns his most preferred object. Since each person points, there is at least one "top cycle:" a group of people, each of whom has the next person's most preferred object and the last of whom has the first person's most preferred object. The algorithm assigns his most preferred object to each member of such a cycle and removes him from the problem. This continues until no one is left.

There are two questions to answer in generalizing this algorithm to deal with indifference. First, when can people be removed from the problem? We formulate a condition that is akin to equating demand and supply. Once a group of people trades (as a part of a cycle), we check if what they hold (think of this as supply) is exactly the set of objects that they most prefer (think of this as demand). If not, they may be able to improve the welfare of someone outside of the group and we keep them in the problem.

Second, who should a person point at if he is indifferent between what two different people hold? To answer this, we first note that only a person who does not hold one of his most preferred objects can be made any better off than he already is. Such a person is "unsatisfied." The rest of the people are "satisfied." When a person is directly or indirectly pointed at, this increases the possible trades available to him. The objective is to increase the trades available to *unsatisfied* people. We define a recursive way of determining whom each person points at. First, anyone who can point at an *unsatisfied* person does so, breaking ties with a fixed priority order. Next, anyone who can point at an *unsatisfied* person *through* a *satisfied* person does so. These people break ties with the same fixed priority order over the eventual *unsatisfied* person that they would reach. Then, we consider anyone who can point at an *unsatisfied* person through two *satisfied* people, and so on.