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Finite-order type spaces and applications ☆

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Abstract

We propose a framework of consistent finite-order priors to facilitate the incorporation of higher-order uncertainties into Bayesian game analysis, without invoking the concept of a universal type space. Several recent models, which give rise to stunning results with higher-order uncertainties, turn out to operate with certain consistent order-2 priors. We introduce canonical representations of consistent finite-order priors, which we apply to establish a criterion for determining the orders of strategically relevant beliefs for abstract Harsanyi type spaces. We derive finite-order projections of type spaces and discuss convergence of BNEs based on them as the projection order increases. Finally, we introduce finite-order total variation distances between priors, which are suitable for analyzing the issues on equilibrium continuity and robustness. We revisit recent advancements of Bayesian game theory and develop new insights based on our framework.

JEL classification: C72; D82

Keywords: Bayesian game; Common prior; Complete information; Consistent prior; Finite-order type space; Payoff type space; Projection type space; Robustness; Total variation distance

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1. Introduction

Popular applications of Bayesian games assume that the players' types are their payoff types and it is common knowledge that their payoff types are jointly drawn by nature according to some common prior. The resulting type space is known as a *payoff type space*, which is the smallest type space we can work with [2, p. 1773]. There is an extreme implication with a payoff type space: each payoff type of a player uniquely determines his belief about the other players' types. Despite its virtue of simplicity, payoff type spaces are restrictive for certain applications. For example, consider a sealed-bid, first-price auction for a high-tech patent between two potential buyers. In addition to the uncertainty about each other's reservation prices (payoff types), one bidder might have reason to suspect that the other bidder can acquire information about his reservation price with some positive probability, say, by hiring a super-talented computer hacker. Payoff type spaces cannot deal with such information settings.

There is, however, a great deal of additional complexity attached to type spaces beyond payoff type spaces. Though they may look simple in the implicit model *a la* Harsanyi [18], their correspondences in Mertens and Zamir's [26] explicit model of *universal type spaces* can be very complicated. Given a Bayesian game, neither the implicit nor the explicit model provides any criterion for determining the upper bounds on the orders of strategically relevant beliefs. As we discuss later in this paper, characterizing type spaces in terms of these upper bounds turns out to be crucial in applications. We introduce the concepts of *consistent finite-order priors* and their *canonical type spaces*, which we apply to explicitly model type spaces with finite-order belief hierarchies without invoking the concept of a universal type space, and to determine the upper bounds on the orders of strategically relevant beliefs for abstract type spaces. We also introduce *finite-order projections* of an abstract type space and *finite-order total variation* distances between priors, which we apply to develop new insights into an array of theory and applied problems associated with higher-order uncertainties.

Our analysis begins with the private-value order-2 common prior case. In this case, nature draws for each player a payoff type and a first-order belief about the others' payoff types according to a common prior. Such a common prior is of order-2 by virtue of being a probability measure over payoff types and first-order beliefs. But, unlike with types in payoff type spaces which are de facto of order 1, nature may draw *multiple* first-order beliefs for a given payoff type of a player. The consistency of the prior then requires that for each player, each of his first-order beliefs drawn by nature coincides with the posterior belief he can derive by updating the common prior using his private information (the payoff type and first-order belief).[1] We show that given a payoff environment, consistent order-2 priors form a convex class that contains all order-1 and complete information priors as proper subsets. It follows that the class of consistent order-2 priors is richer than both the class of order-1 and the class of complete information priors. Indeed, as we discuss later, substantially different results can be established for several familiar Bayesian games once the information structure changes from an order-1 or complete information common prior to a consistent order-2 common prior.

The model of a consistent order-2 prior with private value can be extended to the model of a consistent order-k prior with common value for any finite positive integer k: nature draws a payoff relevant parameter and for each player, a *coherent* belief hierarchy of order $k - 1$. As

[1] For reasons that will become clear after Theorem 1, we use the terms consistent priors and common-prior type spaces interchangeably.