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Knightian decision theory and econometric inferences

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Abstract

An uncertainty averse Knightian decision maker has a set of probability distributions over outcomes and chooses something other than the status quo only if the change increases the expected payoff according to all the distributions. It is possible to define a standardized degree of uncertainty aversion. To each such degree, there corresponds a set of prior distributions over the parameters of a Gaussian linear regression model, these priors being centered on a uniform prior. The set of posterior means corresponding to this set of priors has the same properties as a standard confidence region.

JEL classification: C11; C18; D01

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1. Introduction

In previous papers [4,5], a Knightian theory of decision was proposed in which people compare alternatives using many subjective probability distributions. The multiplicity of the set of subjective distributions arises from ignorance of the true probabilities and aversion to uncertainty. As in the work of Frank Knight [9], the word "uncertainty" is taken to mean random variation according to an unknown probability law, and the word "risk" is applied to random variation according to a known law. Uncertainty aversion is distinct from risk aversion. An increase in uncertainty aversion increases the size of the set of subjective distributions. An uncertainty neutral decision maker has only one subjective distribution and so is Bayesian.

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In this paper, I argue that classical econometric practice may be interpreted as expressing a Knightian decision theoretic point of view, even though formal justifications of classical methods use the frequentist rather than the subjective notion of probability. Classical confidence regions may be viewed as defining sets of posterior means corresponding to a standardized set of prior distributions. Tests of the null hypothesis that a parameter equals a particular value may be viewed as determining whether it is rational, from a Knightian point of view, to act as if the null hypothesis were true. This interpretation of the tests seems to correspond fairly well to practice and to the informal stories told by classical statisticians. One might even argue that classical statisticians act unconsciously as Knightian decision makers. If one accepts this argument, then it is of interest to know what level of uncertainty aversion corresponds to the popular 5% significance level. The definitions of standardized sets of prior distributions make possible a quantifiable form of uncertainty aversion, and the equivalence of classical confidence regions and sets of posterior means establishes a correspondence between confidence levels and these quantifiable levels of risk aversion. In examples, the levels of risk aversion corresponding to the 95% confidence level are quite high, a fact that is perhaps evidence that uncertainty aversion is a real and important aspect of reality.

Others have had ideas similar to those expressed in this paper. Edward Learner [10] argues that probability intervals should be used to describe uncertainty about prior and posterior probabilities and he emphasizes the connection between this Knightian view and his analysis of the sensitivity of posterior to prior distributions. Peter Walley [11] advocates the use of multiple subjective probabilities as a basis for statistical inference and decision theory.

2. Review of Knightian decision theory

The basic ideas and notation of the Knightian decision theory of Bewley [4,5] are briefly as follows. The starting point is a relation of strict preference, $\succ$, defined over a topological vector space, X, of lotteries. The payoffs of the lotteries should be thought of as in utility, or it should be imagined that utility is an affine function of the payoffs. The space X consists of measurable real-valued functions on a state space S with the σ-field of measurable sets $\mathcal{S}$. The topology of X is locally convex and the dual space, X^*, of continuous linear functionals on X may be identified with the set of signed measures on $\mathcal{S}$. I assume that X contains the function that is everywhere equal to 1, so that all the measures in X^* are bounded. The ordering $\succ$ is assumed to be transitive, but may not be the strict preference part of a complete ordering. The ordering $\succ$ is also assumed to be continuous and to satisfy a standard substitution condition. Under these assumptions, $\succ$, may be characterized by a convex and closed set, $\Pi(\succ)$, of measures on $\mathcal{S}$ as follows: $x \succ y$ if and only if $\int (x(s) - y(s))\pi(ds) > 0$, for all $\pi \in \Pi(\succ)$, where $\int f(s)\pi(ds)$ is the indefinite integral of π with respect to π. Depending on context, $\Pi(\succ)$ may be either a cone of non-negative measures or a set of probability measures, but in this paper it is always a set of probabilities. This characterization is established by noting that $C(\succ) = \{x \in X \mid x \succ 0\}$ is a convex cone containing the positive cone of X. The set $\Pi(\succ)$ is dual to $C(\succ)$. The proofs are adapted from arguments of Aumann [1–3].

An additional assumption guarantees that if the probability of an event is known objectively, then all the subjective probability distributions assign it the objective probability, so that the theory does not contradict the von Neumann–Morgenstern theory of choice under risk. Incompleteness of preferences is what distinguishes behavior toward uncertainty from behavior toward risk.