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Scalarization methods and expected multi-utility representations

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Abstract

I characterize the class of (possibly incomplete) preference relations over lotteries which can be represented by a compact set of (continuous) expected utility functions that preserve both indifferences and strict preferences. This finding contrasts with the representation theorem of Dubra, Maccheroni and Ok [16] which typically delivers some functions which do not respect strict preferences. For a preference relation of the sort that I consider in this paper, my representation theorem reduces the problem of recovering the associated choice correspondence over convex sets of lotteries to a scalar-valued, parametric optimization exercise. By utilizing this scalarization method, I also provide characterizations of some solution concepts. Most notably, I show that in an otherwise standard game with incomplete preferences, the collection of *pure* strategy equilibria that one can find using this scalarization method corresponds to a refinement of the notion of Nash equilibrium that requires the (deterministic) action of each player be not worse than any *mixed* strategy that she can follow, given others' actions.

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1. Introduction

Starting with Aumann [5], early research on representation of incomplete preference relations under risk explored sufficient conditions that allow one to extend a preference relation by a *single* expected utility function. Put precisely, given a (possibly incomplete) preference relation $\succsim$ over the set of lotteries on a prize space X, the purpose of a typical work in this early literature is to find a von Neumann–Morgenstern function u such that

$$p \succ q \text{ implies } \int_X u\, dp > \int_X u\, dq, \quad \text{and} \quad p \sim q \text{ implies } \int_X u\, dp = \int_X u\, dq. \tag{1}$$

As also noted by Aumann [5, p. 448], the main merit of this representation notion is that maximization of such an expected utility function over a set will always lead to a maximal lottery in that set.[1] Thereby, in every choice set, we can identify *a* lottery that the decision maker in question can possibly select from that set.

However, when studying economic phenomena related to indecisiveness, the researcher often needs to recover the choice correspondence induced by an incomplete preference relation in its entirety. Indeed, the best-known behavioral consequences of indecisiveness include (i) a certain degree of randomness in choices, which, as Mandler [33] notes, may reflect itself with intransitivity of observed choice behavior; and (ii) the multiplicity of alternatives that might be chosen in a given situation, which is the focus of Rigotti and Shannon [44] in their work on indeterminacy of equilibria in security markets. The study of how an agent may or should resolve her indecisiveness is a related area of research.[2] Moreover, it has been recently observed that a variety of interesting behavioral phenomena can be explained by two-stage choice procedures where in the first stage the agent identifies a *collection* of maximal alternatives in a given choice set (with respect to an endogenously determined incomplete preference relation), and then makes her final choice among these maximal alternatives according to a secondary criterion.[3]

The problem of recovering the choice correspondence induced by an incomplete preference relation gave rise to the literature on multi-utility representations which provide a set of utility functions that fully characterize a given preference relation. In fact, it seems fair to argue that the virtue of such a representation theorem lies in its potential use as an analytical tool that can facilitate the exercise of identifying the choice correspondence associated with a preference relation which satisfies certain behavioral axioms. Naturally, the performance of a representation

[1] In fact, only the first part of property (1) is crucial for this conclusion.

[2] For example, Ok, Ortoleva and Riella [41] propose a model in which the choice between two incomparable alternatives, say x and y, depends on other options in a certain way: the presence of a third alternative z that is asymmetrically dominated by x or y increases the decision maker's tendency to choose the dominating alternative. In turn, Danan [13] studies the problem of "how to choose in the absence of preference" from a normative point of view.

[3] Various reference-dependent choice models, for instance, *necessitate* the use of incomplete preferences in such a procedural context [36,3]. Another example is the procedural model of Manzini and Mariotti [34] that accounts for intransitive choice behavior. A longer list of indecisiveness-related phenomena includes preference for flexibility and choice deferral [15,29], preference for commitment [14], and several implications for political games [45,30].