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Top trading with fixed tie-breaking in markets with indivisible goods ☆

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Abstract

We study markets with indivisible goods where monetary compensations are not possible. Each individual is endowed with an object and a preference relation over all objects. When preferences are strict, Gale's top trading cycles algorithm finds the unique core allocation. When preferences are not necessarily strict, we use an exogenous profile of tie-breakers to resolve any ties in individuals' preferences and apply Gale's top trading cycles algorithm for the resulting profile of strict preferences. We provide a foundation of these simple extensions of Gale's top trading cycles algorithm from strict preferences to weak preferences. We show that Gale's top trading cycles algorithm with fixed tie-breaking is characterized by *individual rationality*, *strategy-proofness*, *weak efficiency*, *non-bossiness*, and *consistency*. Our result supports the common practice in applications to break ties in weak preferences using some fixed exogenous criteria and then to use a "good and simple" rule for the resulting strict preferences.

JEL classification: C78; D63; D70

Keywords: Indivisible goods; Top trading cycles; Indifferences; Fixed tie-breaking

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1. Introduction

Consider the problem of exchanging a number of indivisible goods ("houses" or "objects") among a group of individuals without monetary compensations. Each individual is endowed with one object and consumes exactly one object. The individuals do not pay for the objects; neither is any form of side payments between the individuals permitted. Clearly, this type of problem arises when reallocating courses among professors, apartments among existing tenants (where rents are regulated), offices among employees, tasks among workers, seats at public schools among students, and time sharing slots of vacation apartments among their owners.[1]

Another real-life application is referred to as kidney exchange (Roth, Sönmez and Ünver [24]) whereby patients with their donors would like to receive a compatible kidney. Any paired patient–donor wishes to trade in an exchange of a better kidney donor. Here preferences may be based on checklists of criteria (like blood and tissue types for organ transplant) and distinct objects with the same criteria are equivalent. Hence, both not all indifferences may be possible and two agents may view different objects indifferent. Our main result will be true for those environments as well.

The classical piece by Shapley and Scarf [26] introduced this problem and uses the notion of Edgeworthian exchange to solve it. The individuals are free to engage in multilateral negotiations and exchange by using their allocation either individually or as part of a coalition to block proposed allocations. The set of unblocked allocations is the core. In their classic paper Shapley and Scarf [26] use Gale's "Top Trading Cycle" algorithm to establish the existence of the core of this "housing market" (or market with indivisible goods). A number of subsequent "early" papers have used this "exchange based" approach to investigate further the properties of cores of markets with indivisibilities (see [23,34,17]) showing that in the special case when preferences are restricted to be linear orderings, the core exists, is unique and efficient, and corresponds to the unique competitive allocation of the housing market.

In the housing market with strict preferences, only very few characterizations were obtained. An important paper by Ma [17] showed that the top trading cycles algorithm is the only mechanism satisfying *individual rationality*, *strategy-proofness*[2] (meaning that no individual can gain by misreporting his preference), and *efficiency*. Miyagawa [19] showed that any mechanism satisfying *individual rationality*, *strategy-proofness*, *non-bossiness* ([25]; meaning that no individual can change the allocation without changing his assigned house), and *anonymity* must be either the top trading cycles algorithm or the no-trade mechanism. However, so far no characterization has been obtained in housing markets with indifferences. In most of the literature on house allocation, agents are assumed to possess asymmetric, also called strict, preferences over the set of houses. In applications we cannot rule out that agents may be indifferent between various objects. It is important to understand this case.

Using intuitive properties we characterize (in terms of welfare-equivalence) a set of rules which is frequently used in real life: given a profile of weak preferences we break ties using some exogenously fixed tie-breakers. Then for the obtained strict profile we simply apply Gale's top trading cycles algorithm. We take as a basic set of axioms *individual rationality*, *strategy-proofness*, and *non-bossiness*. By Jaramillo and Manjunath [12, Proposition 2] it is impossible for any such rule to be *efficient* and we can at most require *weak efficiency*: there is no allocation

[1] The introduction in [20] gives examples of other similar situations.

[2] Roth [22] was the first to show that for strict preferences the top trading cycles algorithm satisfies *strategy-proofness*.