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Strategy-proof stochastic assignment

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Abstract

I study strategy-proof assignment mechanisms where the agents reveal their preference rankings over the available objects. A stochastic mechanism returns lotteries over deterministic assignments, and mechanisms are compared according to first-order stochastic dominance.

I show that non-wasteful strategy-proof mechanisms are not dominated by strategy-proof mechanisms, however non-wastefulness is highly restrictive when the mechanism involves randomization. In fact, the Random Priority mechanism (i.e., the Random Serial Dictatorship), and a recently adopted school choice mechanism, Deferred Acceptance with Random Tie-breaking, are wasteful. I find that both these mechanisms are dominated by strategy-proof mechanisms.

In general, strategy-proof improvement cannot be due to merely reshuffling objects, and therefore must involve assigning more objects.

JEL classification: C78; D61; D63

Keywords: Random assignment; Strategy-proofness; Priority based assignment; Ordinal efficiency; School choice

1. Introduction

Consider the problem of assigning a number of heterogenous indivisible objects to individuals, where each individual can receive at most one object. In a wide range of contexts where this problem arises, such as school choice, course assignment at universities, or office allocation, the procedures typically rule out monetary transfers, and rely on agents' preference rankings over the objects. Three properties are crucial for an assignment procedure: efficiency, strategy-proofness, and fairness.

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Indivisibilities make it impossible to treat the agents equally when assignments are made in a deterministic way. *Randomization* greatly expands the set of mechanisms and allows *fairness*. For example, the *random priority mechanism* orders the agents randomly, and lets them pick their objects in this order. In the richer set of stochastic mechanisms, agents' assignments are lotteries over objects. The assignment at the end of the procedure in application will depend on the realization of the lottery, but the agents, when revealing their preferences, are facing lotteries over deterministic outcomes. Therefore, the analysis of the mechanism calls for a stochastic perspective.

Since the agents' preferences are their private information, it is important for the mechanism to provide the agents with the right incentives to reveal their preferences truthfully. Thus efficiency is really a property of the mechanism, rather than of the assignment resulting from the mechanism after the preferences are revealed. In our environment the primitives on preferences are ordinal rankings over the objects, so the appropriate concept for incentive compatibility is *strategy-proofness*, and the natural notion of welfare is *Pareto efficiency*, where lotteries are compared via *first-order stochastic dominance*.

A mechanism *weakly dominates* another if for every preference profile, it assigns to every agent a lottery which she weakly prefers in the first-order stochastic dominance sense. As it is with efficiency, *non-wastefulness* is also extended to this stochastic environment in a natural way: if an agent would rather have more of some object, say x, instead of another object she has received with positive probability, then it must be that all of x is already assigned.

I first show that a non-wasteful strategy-proof mechanism cannot be dominated by another strategy-proof mechanism. On the other hand, for randomized mechanisms, this *ordinal*, or *ex ante*, notion of non-wastefulness is a very demanding condition, much stronger than *ex post* non-wastefulness. While every non-wasteful stochastic assignment is a convex combination of non-wasteful deterministic assignments, stochastic mechanisms are often wasteful even when they are randomizations over non-wasteful mechanisms.

My next result, then, sheds light on the nature of strategy-proof improvement: if a strategy-proof mechanism is dominated by another, such Pareto improvement cannot be achieved by merely re-allocating the objects, but must involve assigning *more* objects. Indeed, in two important applications, I show that the *random priority* (*RP*), i.e., the *random serial dictatorship* mechanism, and the *randomized deferred acceptance* (*RDA*) mechanism admit strategy-proof improvement. I give explicit constructions of strategy-proof mechanisms which dominate them.

Bogomolnaia and Moulin [3] make the critical observation that the RP is dominated, but they also show that there is no strategy-proof and efficient mechanism satisfying equal treatment of equals. Zhou [23] notes that whether the RP is optimal within the class of symmetric, ex post Pareto optimal, strategy-proof mechanisms remains an open question. My construction shows that it is not.

A question of the same nature recently emerged from another practical market design issue. In a typical school choice program, each school has an exogenous priority ranking over the students. A matching *respects these priorities* if whenever a student prefers a school x to her own, it must be that only students of equal or higher priority for x are assigned to school x. When these priority rankings are strict, the *Deferred Acceptance* (*DA*) mechanism of Gale and Shapley [12] returns the *constrained efficient* assignment (i.e., the student-optimal assignment among those assignments which respect the priorities). However, in many applications, large groups of students have equal priority, and therefore the priority rankings have ties. The leading mechanism in this environment *randomly* breaks ties before applying the DA mechanism. Ehlers [8] observes that some tie-breaking rules even result in constrained inefficiency. Therefore, unlike