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Uniqueness of equilibrium in directed search models ☆

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Abstract

We study a decentralized trading model as in [7], where a finite number of heterogeneous capacity-constrained sellers compete for a finite number of homogeneous buyers, by posting prices. This "directed search" model is known to admit symmetric equilibria; yet, uniqueness has proved elusive. This study makes two contributions: a substantive contribution is to establish uniqueness of symmetric equilibrium; a methodological contribution is to develop a tool based on directional derivatives to characterize equilibrium.

JEL classification: C70; D390; D490; E390

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1. Introduction

This study concerns equilibrium in finite markets where heterogeneous sellers compete for homogeneous buyers by posting prices. The central characteristic of the economy is that market participants face a trade-off between the price posted and the probability of trading; see [1,6–8].

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More precisely, the market is composed of a finite population of capacity-constrained sellers and mobility-constrained buyers. Sellers are endowed with (or can produce) one homogeneous good; consumption of this good—usually assumed indivisible—gives utility to buyers who can, in turn, transfer utility to sellers. The market is modeled as a sequential game of complete information where sellers take the lead by simultaneously and independently posting (and committing to) a price, i.e., a utility level for any buyer who wishes to trade. Buyers see all prices, and simultaneously and independently visit one seller, i.e., they can direct their search. Once meetings occur, sellers trade at the posted price satisfying capacity constraints through random rationing.

Models of this type have been adopted to tackle a variety of issues in labor and IO, such as wage and price dispersion, market efficiency, and competing mechanisms; see [3,5,9] for some recent examples. Many equilibria exists in this setting, some of which are symmetric and some of which are not; see the examples in [1]. The focus in this literature has been (strongly) symmetric equilibrium, existence of which is established in [4,7]. A significant open question is whether symmetric equilibrium is unique. Establishing uniqueness has so far proved elusive because of the analytical intractability associated to working with finite numbers of buyers and heterogeneous sellers; see [8]. In fact, most studies focus on limit economies where sellers are homogeneous and the number of players gets large. Our study fills this important theoretical gap in the literature on markets where search can be directed.

The analysis we conduct provides two contributions. A substantive contribution is to present a theorem establishing uniqueness of symmetric equilibrium in directed search economies with finite numbers of players and heterogeneous sellers such as those in [1,6–8]. The result can be extended to economies with infinite, heterogeneous populations. The analysis provides a methodological contribution, also. In the type of markets we study global concavity is not a property of sellers' payoff functions; therefore, standard methods of analysis cannot be applied to determine uniqueness. We develop a tool to handle this type of situation; the technique is based on use of directional derivatives along judiciously selected equilibrium "price paths."

The paper proceeds as follows. Section 2 presents the model and lays out some notation. Section 3 offers some preliminaries involving properties of demand. Section 4 contains the main results and Section 5 concludes.

2. The model

Consider an economy with finitely many sellers and buyers. Buyers are homogeneous, but sellers could be heterogeneous. Let $\mathcal{J} = \{1, \ldots, \bar{J}\}$, $2 \leqslant \bar{J} < \infty$ be the set of sellers, and $\mathcal{I} = \{1, \ldots, I\}$, $2 \leqslant I < \infty$ be the set of buyers. Each seller has one indivisible good which cannot be consumed by sellers, but are desirable to buyers.

Buyers and sellers play a sequential game of complete information, over three stages. In the first stage, sellers simultaneously and independently post and commit to a price for the indivisible good they sell. Let v_j be the (indirect) utility for a buyer who purchases the good offered by seller j at the price posted by that seller. Hence, $\mathbf{v} = (v_1, \ldots, v_{\bar{J}}) \in \mathbb{R}_+^{\bar{J}}$ is the strategy profile of all sellers and $\mathbf{v}_{-j}$ denotes the strategy profile of every other seller, when we fix seller j. Denote the set of all *feasible* promised utilities as

$$\mathcal{V} := \prod_{j \in \mathcal{J}} [\underline{v}_j, \bar{v}_j] \subset \mathbb{R}_+^{\bar{J}}, \quad \text{where } 0 \leqslant \underline{v}_j < \bar{v}_j \text{ for all } j \in \mathcal{J}.$$