



The Condorcet set: Majority voting over interconnected propositions [☆]

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Abstract

Judgement aggregation is a model of social choice in which the space of social alternatives is the set of consistent evaluations ('views') on a family of logically interconnected propositions, or yes/no issues. However, simply complying with the majority opinion in each issue often yields a logically inconsistent collective view. Thus, we consider the *Condorcet set*: the set of logically consistent views which agree with the majority on a maximal subset of issues. The elements of this set turn out to be exactly those that can be obtained through *sequential majority voting*, according to which issues are sequentially decided by simple majority unless earlier choices logically force the opposite decision. We investigate the size and structure of the Condorcet set for several important classes of judgement aggregation problems. While the Condorcet set verifies a version of McKelvey's (1979) celebrated 'chaos theorem' in a number of contexts, in others it is shown to be very regular and well-behaved.

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In the context of preference aggregation, pairwise majority voting often fails to produce unambiguous outcomes because a Condorcet winner might not exist. Similarly, in the general judgement aggregation problem where each social outcome is described as a pattern of yes/no answers to certain interrelated propositions (a ‘view’), issue-wise majority voting may fail to produce consistent collective ‘views;’ see Guilbaud [14] and Kornhauser and Sager [16]. Consistency is guaranteed only in those cases in which the aggregation problem has the structure of a median space; see Barthélemy and Monjardet [4] and Nehring and Puppe [33].

Does this mean that the very idea of majoritarian aggregation is ill-defined outside median spaces? Does ‘inconsistency’ imply ‘impossibility’? One can arrive at this conclusion from a reading of the Arrovian tradition in social theory and judgement aggregation, especially if one takes the well-known issue-wise independence condition as non-negotiable. William H. Riker, for example, has influentially argued that, “when no Condorcet winner exists, majority rule is theoretically confused and practically inadequate” [36, p. xiii].

But, we submit, such a pessimistic conclusion need not be warranted. Rather than looking for ‘the’ uniquely defined majority view, it is sensible to be more modest and look for views that are at least adequate from a majoritarian perspective. In this vein, for example, Hervé Moulin has suggested that “a Condorcet consistent voting rule chooses a way to live with the paradox: when a Condorcet winner does not exist, it picks a reasonable substitute” [29, p. 231]. Yet, apart from the discussion of the pros and cons of specific properties of different voting rules, there has been surprisingly little discussion in the literature of what exactly should count as a ‘reasonable substitute’ in this context. Here, we provide such a criterion, Condorcet admissibility. Equipped with this criterion, a majoritarian can sidestep impossibility, but may have to live with indeterminacy.

As indicated, our analysis is set in a judgement aggregation framework. A collection of yes/no answers to all propositions is called a *view*; a view is *consistent* if it respects the logical interrelations between issues. The set of all possible consistent views is called an *aggregation space*. A consistent view is *more representative* than another consistent view if its judgement on every issue is supported by at least as many voters, and on some issues by strictly more voters. A consistent view is *Condorcet admissible* if there is no other consistent view that is more representative. The set of such views is the *Condorcet set*. It is a singleton if and only if the issue-wise majority view is itself consistent, in which case it coincides with that view. The Condorcet set comprises exactly those views that can count as ‘majoritarian,’ i.e. that have some minimally defensible majoritarian rationale. In this manner, we offer a rebuttal of Riker’s claim that majority rule is “theoretically confused.” “Practical adequacy” is another matter, and in view of the pervasive impossibility results in social choice theory, indeed a challenging problem; we do not attempt to resolve this problem in the present paper.

The specific task of this paper is to investigate the properties of the Condorcet set, considered as a solution concept for judgement aggregation. A focal question concerns the *size* of the Condorcet set, and thus the extent of majoritarian indeterminacy. We show that, frequently, the Condorcet set can be ‘large.’ In particular, we focus on the question of when the Condorcet set at a given profile of individual views contains, for *every* proposition, both a view that affirms that particular proposition and a view that negates it. We refer to such profiles as *globally indeterminate*, and to aggregation spaces as *globally indeterminate* if they admit globally indeterminate

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