Available online at www.sciencedirect.com

ScienceDirect

Journal of Economic Theory 151 (2014) 411–447

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Stationary Markov perfect equilibria in risk sensitive stochastic overlapping generations models ☆

Anna Jaśkiewicz [a], Andrzej S. Nowak [b,*]

[a] *Institute of Mathematics and Computer Science, Wrocław University of Technology, Poland*
[b] *Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, Poland*

Received 11 April 2013; final version received 29 November 2013; accepted 26 January 2014

Available online 31 January 2014

Abstract

In this paper, we study intergenerational stochastic games that can be viewed as a special class of overlapping generations models under uncertainty. Making use of the theorem of Dvoretzky, Wald and Wolfowitz [27] from the statistical decision theory, we obtain new results on stationary Markov perfect equilibria for the aforementioned games, with a general state space, satisfying rather mild continuity and compactness conditions. A novel feature of our approach is the fact that we consider risk averse generations in the sense that they aggregate partial utilities using an exponential function. As a byproduct, we also provide a new existence theorem for intergenerational stochastic game within the standard framework where the aggregator is linear. Our assumptions imposed on the transition probability and utility functions allow to embrace a pretty large class of intergenerational stochastic games analysed recently in macroeconomics. Finally, we formulate a set of assumptions under which the stochastic process induced by the stationary Markov perfect equilibrium possesses an invariant distribution.

JEL classification: C62; C73; D91; O40

Keywords: Overlapping generations model; Intergenerational stochastic game; Risk sensitive optimisation; Stationary Markov perfect equilibrium

☆ This work is partially supported by the National Science Centre: Grant DEC-2011/03/B/ST1/00325.
* Corresponding author.
E-mail addresses: anna.jaskiewicz@pwr.wroc.pl (A. Jaśkiewicz), a.nowak@wmie.uz.zgora.pl (A.S. Nowak).

0022-0531/$ – see front matter
http://dx.doi.org/10.1016/j.jet.2014.01.005

1. Introduction

Intertemporal decision making that utilises standard optimisation criteria such as the total or discounted expected return over the time horizon may be quite insufficient to characterise the problem from the point of view of the decision maker. This is because these measures do not reflect the variability-risk features of the model. In order to circumvent the difficulty Howard and Matheson [47] studied a Markovian controlled model, in which the decision maker is equipped with a constant absolute risk coefficient. This assumption, in turn, implies that such a decision maker grades his/her utility via the expectations of the exponential function of future random outcomes. Moreover, the Taylor expansion of the exponential function allows to observe that such a decision maker does not only take into account the expectation of random returns received in the future, but also he/she values (with different weights) all their higher moments. This fact resulted in a rapid development of risk sensitive control theory in diverse research areas. In particular, the stimulating ideas have found a number of applications in macroeconomic models, see Tallarini [72], Hansen and Sargent [39], Anderson et al. [5] and references cited therein. The reader interested in further virtues and properties of this criterion is referred, e.g., to the comprehensive monograph of Whittle [75] and Föllmer and Schied [30].

A key feature of the aforementioned works is that they admit a (normative) representative decision maker. In many real life situations, however, the assumption of a representative decision maker is not acceptable. For example, the arrival of new decision makers in the economy is not only realistic but also yields a range of fresh economic interactions. These interactions are neatly captured in various overlapping generations (OLG) models. OLG models are most often used in macroeconomics, but they can also be useful in microeconomics (see, for example, Ljungquist and Sargent [54], Bewley [12], Acemoglu [1] or Geanakoplos [33], for general accounts). The baseline OLG model and its various deterministic variants gave rise to the extensive study of OLG models within a stochastic framework, see Peled [67], Duffie et al. [24] and references cited therein are good examples of works that deal with this issue. Specifically, Duffie et al. [24], making use of certain ideas from the theory of Markov processes on Borel state space and stochastic games, proved a general theorem on the existence of stationary Markov equilibrium that induces an ergodic process in an OLG model. Similar results on the existence of Markov perfect equilibrium for different OLG models were given in Gottardi [35] and Harris and Laibson [40], Krusell and Smith [51]. It is worthy to observe that although the latter two works are formulated in terms of "hyperbolic consumers" rather than generations, their results can be easily expressed for a certain type of an OLG model. A common feature of all aforementioned papers on stochastic OLG models is that they use additive aggregation of partial utilities concerning actions of following generations.

A major contribution of this paper is an application of principle ideas from risk sensitive control theory to certain classes of OLG models. To the best of our knowledge, such a merging of the two theories has not been encountered so far in the literature.

In this paper, we are concerned with a stochastic intergenerational bequest game inspired by the seminal work of Phelps and Pollak [68]. In their model, it is assumed that each generation lives, saves and consumes over just one period. Moreover, each generation cares about the consumption of the following generations, in the sense that it wants to leave a bequest to the successors. Therefore, such a generation derives utility from its own consumption and those of its descendants. The next generation's inheritance or capital is described by a production function that is linear with respect to the invested capital. The various versions of this model were studied by numerous authors. For instance, Leininger [52] and Bernheim and Ray [14] independently