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Subjective multi-prior probability: A representation of a partial likelihood relation

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Abstract

This paper deals with an incomplete relation over events. Such a relation naturally arises when likelihood estimations are required within environments that involve ambiguity, and in situations which engage multiple assessments and disagreement among individuals' beliefs. The paper characterizes binary relations over events, interpreted as likelihood relations, that can be represented by a unanimity rule applied to a set of prior probabilities. According to this representation an event is at least as likely as another if and only if there is a consensus among all the priors that this is indeed the case. A key axiom employed is a cancellation condition, which is a simple extension of similar conditions that appear in the literature.

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Keywords: Multi-prior probability; Incomplete relation; Ambiguity; Cancellation

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1. Introduction

1.1. Motivation and background

Estimating the odds and comparing the likelihood of various events are essential parts of processes carried out by many organizations. For instance, the US intelligence community produces National Intelligence Estimates, in which the likelihood of various events is assessed. Questions such as, 'Is it more likely that democracy will prevail in Libya, or that a military regime will be established?', and the like, seem natural to ask. These questions and many others call for likelihood comparisons of different events. Other examples in which probabilistic estimates are used include forecasts published by central banks, that address issues such as the odds of inflation or recession, estimated likelihood of natural events such as global warming, that are based on individual opinions of scientists and on many experiments, and so forth. In all these instances, statements of the type 'event A is more likely than event B' seem fundamental to the respective context.

In many situations, assessments of the kind given above are a product of advisory entities (e.g. intelligence analysts, consulting firms, scientists), that output likelihood judgements and forward them to a decision maker, who in turn takes these into account when making a decision. Typically, such assessments rely on 'objective' data, such as reports of military movements, temperature measurements and the like, and are intended to be based as closely as possible on the data. Frequently, though, the events examined involve some degree of ambiguity. Knowledge or available information might be insufficient to determine which of two events under consideration is more likely, and a likelihood relation in such situations might therefore leave the comparison between some pairs of events unspecified.

Motivated by the above question, this paper characterizes a likelihood relation that may be incomplete. The question of representing an incomplete likelihood relation was already addressed is Nehring [17]. In his paper, Nehring considers an incomplete 'at least as likely as' relation over events, and proposes to represent it using a consensus rule over a set of prior probabilities.[2] His result matches the results of Giron and Rios [10], Bewley [2], and the purely subjective work of Ghirardato et al. [9] – all being characterizations of multi-prior expected utility representations – to the domain of comparisons between events. Our paper is closely related to Nehring [17]. Similarly, it formulates conditions on a binary relation over events, that are necessary and sufficient for the relation to be represented by a consensus rule over a set of prior probabilities.

Formally, let $\succsim$ denote a binary relation over events, where $A \succsim B$ for events A and B is interpreted as 'A is at least as likely as B'. The main theorem of the paper introduces necessary and sufficient conditions ('axioms') on $\succsim$ that guarantee the existence of a set of prior probabilities, $\mathcal{P}$, such that for any two events A and B,

$$A \succsim B \quad \Leftrightarrow \quad \mu(A) \geqslant \mu(B) \quad \text{for all } \mu \in \mathcal{P}. \tag{1}$$

The difference between the representation theorem in this paper and that in [17] lies in the axioms assumed, most notably a richness assumption used in [17] that we replace in the case of an infinite state space, and drop completely when finite state spaces are concerned. As a result, the paper contains what we consider to be a purely subjective treatment of partial likelihood

[2] Nehring then pairs this likelihood relation with a preference relation and explores the compatibility between the two relations. This paper focuses on likelihood relations, and does not investigate preference relations.