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An inequality measure for stochastic allocations ☆

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Abstract

Few papers in the literature on inequality measurement deal with uncertainty, particularly when the ranking of cohorts may not be fixed. We present a set of axioms implying such a class of inequality measures under uncertainty that is a one-parameter extension of the generalized Gini mean over the distribution of average allocations. The extension consists of a quadratic term accounting for inter-personal correlations. In particular, our measure can simultaneously accommodate a preference for "shared destiny", a preference for probabilistic mixtures over unfair allocations, and a preference for fairness "for sure" over fairness in expectation.

JEL classification: D63; D71; D81

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1. Introduction

Many have attributed the genesis of the modern literature on income inequality measurement to the works of Kolm [28] and Atkinson [3]. For a given social welfare function, they defined a representative level of income $r(x)$ which if distributed equally would give rise to the same

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level of social well being as the given income distribution x. Under the principle of transfer [12,29], the representative income of the income distribution x would always be less than its average income $\bar{x}$ except when societal incomes are distributed equally. This led them to define an inequality measure as $1 - r(x)/\bar{x}$. This class of inequality measures includes the Gini index, arguably the most widely used measure of income inequality, in which the representative income is given by taking the average of a decreasingly arranged distribution of incomes $\{x_1, \ldots, x_N\}$ with $2i - 1$ weight being assigned to the ith richest person [32]. The Gini representative income is then given by

$$\{x_1 + 3x_2 + \cdots + (2N-1)x_N\}/N^2.$$

Using an additive social welfare function based on a power function, Atkinson [3] derived a one-parameter family of inequality indices. In the same year, Rothschild and Stiglitz [30] offered a definition of increasing risk among probability distribution functions. It is noteworthy that their definition of increasing risk mirrors the Pigou–Dalton principle of transfer which underpins much of the inequality measurement literature.

There is increasing recognition of the limitations of earlier inequality measures which, among other things, do not generally incorporate uncertainty.[1] This is illustrated by the following example involving two individuals ($i = 1, 2$) and two equally likely states ($s = 1, 2$). An allocation in state s to individual i can be represented via the 2×2 matrix C_{si}. We seek social preferences over allocation matrices that can exhibit the following properties: For any $u, v, x, y, z \in \mathbb{R}_+$,

$$\begin{pmatrix} x & y \\ u & v \end{pmatrix} \sim \begin{pmatrix} y & x \\ v & u \end{pmatrix} \sim \begin{pmatrix} u & v \\ x & y \end{pmatrix}, \tag{1}$$

$$\underset{A}{\begin{pmatrix} \frac{z}{2} & \frac{z}{2} \\ \frac{z}{2} & \frac{z}{2} \end{pmatrix}} \succcurlyeq \underset{B}{\begin{pmatrix} z & z \\ 0 & 0 \end{pmatrix}} \succ \underset{C}{\begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}} \succ \underset{D}{\begin{pmatrix} z & 0 \\ z & 0 \end{pmatrix}}. \tag{2}$$

The first ranking implies indifference to the permutation of identities and a notion of *state independence* (i.e., indifference to the permutation of state labels given that states are assumed to be equally likely). The second set of rankings correspond to a *weak aversion to aggregate risk* (i.e., $A \succcurlyeq B$), followed by *a preference for shared destiny or ex-post fairness* (i.e., $B \succ C$), further followed by *a preference for ex-ante fairness* (i.e., $C \succ D$). One can view these preferences as being concerned with the type of example given by Diamond [15], where a mother wishes to allocate a good between her two children, and is restricted to an *average* allocation of $\frac{z}{2}$ per child. The mother would most prefer to give each child $\frac{z}{2}$ for sure. If this cannot be achieved, then to avoid envy and the potential for conflict amongst the children, she would prefer that each child receives the same amount in each state (hence, $B \succ C$). The least desirable allocation is the one in which one child is maximally favored for sure. Alternatively, one can view the rankings in (2) as corresponding to two unborn population cohorts who will be endowed with opportunities by the preceding generation. Allocation D corresponds to a situation where cohorts are predestined for their socioeconomic status (as, say, in a rigid class system). Allocation C corresponds to a situation where opportunities are equal for all new generations, but chance alone will ensure that the cohorts will fare unequally. Allocation B corresponds to a situation where opportunities are equal state-by-state, and Allocation A is one where opportunities are equal across individuals and states.

[1] See Chapter 3 of [2] for a discussion of this in the setting of the measurement of socioeconomic mobility.