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Arbitrage-free valuation of interest rate securities under forward curves with stochastic speed and acceleration

Gurupdesh Pandher

Department of Finance, DePaul University, 1 East Jackson Boulevard, Chicago, IL 60604, USA

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Abstract

Arbitrage-free models for valuing interest rate securities posit that stochastic changes in spot or forward interest rates (forward rate "speed") follow a diffusion process. This paper extends the Heath, Jarrow and Morton [Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuations, Econometrica 60 (1992) 77–105], HJM framework by allowing diffusive shocks to both the "speed" and "acceleration" of forward rates. The arbitrage-free restriction on forward rates is identified and involves volatilities of the speed and acceleration dynamics and their correlation. Although the extended forward rates remain in the diffusive framework and evolve continuously, they may exhibit large changes over short intervals (as with jumps) due to stochastic acceleration. Comparisons of bond prices show that the proposed model generates more complex and intricate shapes for the restricted forward curve with the same number of stochastic factors and volatility.

JEL classification: G13; E43; C1

Keywords: HJM; Forward rates; Arbitrage-free restriction; Volatility estimation

1. Introduction

The arbitrage-free approach to valuing interest rate securities and derivatives identifies equilibrium prices that rule out arbitrage against the prevailing yield curve (term structure of interest rates). Heath et al. [9] posit that forward curve stochastics are driven by diffusive changes in forward rates (forward rate "speed") and determine the no-arbitrage restriction on forward rates

E-mail address: gpandher@depaul.edu.

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required for equilibrium pricing. A parallel no-arbitrage framework uses short interest rates to model the term structure (e.g. [10,11,6]). [1] While the short rate formulation uses one state variable to model the term structure, the forward curve formulation uses an infinite state process based on forward rates of differing maturities.

This paper extends the Heath, Jarrow and Morton [9] (HJM hereafter) framework by allowing diffusive shocks to both the "speed" and "acceleration" of forward rates (changes in forward rate drift). Under the extended forward rate dynamics, we identify the restriction on the forward rate process that rules out arbitrage among zero-coupon bonds of different maturities. In addition to the standard HJM forward rate change (speed) volatilities, the arbitrage-free forward rate restriction involves forward rate acceleration volatilities and correlations between the speed and acceleration dynamics. This leads to more complex and realistic shapes for the restricted forward curve with few stochastic factors and has useful empirical and theoretical implications for arbitrage-free interest rate modeling. Another attractive feature of the model is that while the extended forward curve remains in the diffusive framework and evolves continuously, it may exhibit large changes over short intervals (as with jumps) due to diffusive "acceleration".

For the single factor model, the HJM restriction on forward rate drifts is given by $\sigma(t,T)\int_{y=t}^{T}\sigma(t,y)\,dy$ where $\sigma(t,T)$ is the volatility of changes in the forward rate at time t with maturity T. The corresponding restriction on forward rate drifts arising from the diffusive speed and acceleration HJM model (DA-HJM) is

$$\sigma(t,T)\int_{y=t}^{T}\sigma(t,y)\,dy+\int_{u=0}^{t}\delta(u,T)\left[\int_{y=t}^{T}\delta(u,y)\,dy\right]du$$

$$+\rho(t)\left[\sigma(t,T)\int_{y=t}^{T}\delta(t,y)\,dy+\delta(t,T)\int_{y=t}^{T}\sigma(t,y)\,dy\right],$$

where $\sigma(t,T)$ and $\delta(t,T)$ are volatilities of the forward rate speed and acceleration, respectively, and $\rho(t)$ is the correlation between the Brownian motions for the forward rate speed and acceleration.

While the HJM framework has been extended in various ways, the proposed DA-HJM model with diffusive speed and acceleration dynamics in the forward curve has not been considered in the literature. HJM extensions and generalizations include the time-to-maturity formulation of Brace and Musiela [4], the libor market models of Brace et al. [3], Jamshidian [12] and Miltersen et al. [16], the modeling of forward curves as a Gaussian random field by Kennedy [14,15], and the use of stochastic strings to represent the uncertainty of the forward rate curve by Goldstein [8] and Santa-Clara and Sornette [18]. Recently, Johannes [13] and Piazzesi [17] model jumps in short interest rate models triggered by macroeconomic announcements and surprises and find that jumps play a minor role in determining the cross-section of bond prices, but have a greater impact on interest rate option prices.

Obtaining complex shapes of the forward curve and realistic correlations between bonds of different maturities requires a large number of state variables that can make the models difficult to estimate. In the standard HJM model, the same shocks from a finite set of stochastic factors

[1] Ho and Lee [10] model the term structure using short rates and were the first to obtain arbitrage-free values for bonds and options on bonds. Hull and White [11] extend the no-arbitrage framework using the mean-reverting short rate dynamics of Vasicek [20] and Black et al. [1] model short rates using a log-normal process. A general affine class of short rate models is considered by Duffie and Kan [6] and the class is extended to include jumps by Duffee et al. [7].