Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 137 (2007) 696–708
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Notes, Comments, and Letters to the Editor

A tight sufficient condition for Radner–Stiglitz nonconcavity in the value of information

Michel De Lara[a,*], Laurent Gilotte[b,1]

[a] *CERMICS, École des ponts, Paris Tech, 77455 Marne la Vallée Cedex 2, France*
[b] *CIRED, 94736 Nogent-sur-Marne Cedex, France*

Received 14 January 2005; final version received 21 January 2007
Available online 28 March 2007

Abstract

This paper deals with the existence of a nonconcavity in the value of information, as was first explained by Radner and Stiglitz [A nonconcavity in the value of information, in: M. Boyer, R.E. Kihlstrom (Eds.), Bayesian Models in Economic Theory, Elsevier Science Publishers, Amsterdam, 1984, pp. 33–52 (Chapter 3)]. After defining *infinitesimal information distance variation* IIDV, we find that IIDV $= 0$ is sufficient for a zero marginal value of information at the null. This is a condition only on the information structure and in particular is independent of the decision maker's preferences. This condition is tight: when IIDV > 0, there exists a payoff function for which the marginal value of information at the null is positive under general assumptions.

JEL classification: D830

Keywords: Value of information; Marginal value of information at the null; Radner–Stiglitz nonconcavity

1. Introduction

This paper reexamines sufficient conditions for the Radner–Stiglitz nonconcavity of the value of information to a single decision maker. The basic problem is the classic one in which the decision maker chooses an action under uncertainty about a payoff-relevant state of the world. The decision maker has some prior beliefs over possible states and may also observe a realized signal from an information structure. The decision maker chooses a decision rule mapping realized signals

* Corresponding author. Fax: +33 164153586.
E-mail addresses: delara@cermics.enpc.fr (M. De Lara), gilotte@centre-cired.fr (L. Gilotte).
[1] Present address: EDF R&D, B.P. 408, 92141 Clamart Cedex, France.

0022-0531/$ - see front matter © 2007 Published by Elsevier Inc.
doi:10.1016/j.jet.2007.01.014

into actions. The value of any information structure may be defined as the maximum expected payoff that can be achieved using this information structure minus the maximum expected payoff achieved when no signal is available.

In some environments of interest, the decision maker may choose the information structure in advance of the decision problem. A common specification for the set of information structures is, quoting Radner and Stiglitz (RS) in [4], "a family of available information structures indexed by a real parameter, in which larger values of the parameter correspond to more costly information, and in which a zero value of the parameter corresponds to a structure that is both costless and noninformative". The value of information can now be defined as a function of this parameter and its behavior is of interest; for instance, if the value of information minus the cost of information is a globally concave function, then solving the decision maker's information choice problem might be relatively simple.

In a well-known paper [4], RS find conditions under which, for finite state and signal spaces, information has zero marginal value at the null index. If some larger amount of information has a positive net value, this implies that the value function cannot be globally concave in the parameter whenever the marginal cost of information is strictly positive. Chade and Schlee (CS) in their paper [2] provide an overview of economic applications where this nonconcavity has substantive consequences. They extend the RS result in a very general Bayesian framework and clarify the role played by the diverse conditions they provide, which can yield or prevent the result. While conditions in RS beared jointly on information and preferences, CS also first propose some distinct conditions on preferences and on information.

We pursue this effort for identifiable conditions for the RS result and introduce an *infinitesimal information distance variation* (IIDV) which depends only upon the family of information structures and describes how much the revised beliefs depart from the prior when the index goes to zero. When $\text{IIDV} = 0$, we shall prove that the marginal value of information is zero with all bounded payoff functions. But $\text{IIDV} = 0$ is a severe restriction that can be interpreted as equivalent to informativeness being initially at best of second-order. However, we prove in the density case and under mild assumptions that $\text{IIDV} = 0$ is tight in the sense that, if the condition is not satisfied, a payoff function can be found, for which the initial marginal value of information is positive.

The paper is organized as follows. Section 2 sets out briefly the general decision problem and the original RS theorem, together with CS extensions. Section 3 states our main results. Section 4 concludes and points future extensions. All proofs are given in the appendix.

2. Problem statement and previous results

In this section, we recall the original RS theorem, as well as some of the key findings of CS.

2.1. The original economic problem

We consider a decision maker who faces an uncertain state of the world and chooses an action so as to maximize an expected payoff after observing a signal from an information structure.

- The set Ω of the states of the world is a Borel space, endowed with the Borel σ-algebra $\mathcal{B}(\Omega)$ generated by the open subsets (see [1, p. 117]).
- The prior beliefs of the decision maker are represented by a distribution μ on $(\Omega, \mathcal{B}(\Omega))$. We assume that μ has support Ω.
- The decision maker observes a signal in a Borel space $(\mathbb{Y}, \mathcal{B}(\mathbb{Y}))$.