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Recontracting and stochastic stability in cooperative games

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Abstract

An evolutionary style model of recontracting is given which guarantees convergence to core allocations of an underlying cooperative game. Unlike its predecessors in the evolution/learning literature, this is achieved without assumptions of convexity of the characteristic function or a reliance on random errors. The stochastic stability properties of the model are then examined and it is shown that stochastically stable states solve a simple and intuitive minimization problem which reduces to maximizing a Rawlsian SWF for a common class of utility functions. In contrast to previous analyses, the stochastically stable state is unique for a broad class of utility functions.

JEL classification: C71; C72; C73; C78

Keywords: Bargaining; Learning; Core; Convexity; Coalitions

1. Introduction

The papers of Green [1] and Feldman [2] show how a process of coalitional recontracting can converge to core allocations of a cooperative game under specific assumptions. The papers of Agastya [3,4] and Rozen [5] achieve similar results for myopic adaptive processes applied to non-cooperative representations of characteristic function form games: for convex characteristic

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functions the steady states of the processes lie in the core. Framing the model as an adaptive process applied to a non-cooperative game with explicit individual strategies allows the theory of stochastic stability [6,7] to be used to determine which of the steady states of the process is most robust to random errors made by players when choosing their strategies.

This paper unites these two strands of literature by incorporating joint strategic switching into the social learning dynamic which drives strategic change in the non-cooperative representation of any given characteristic function form game. Convergence to interior core allocations is shown for this dynamic under assumptions similar to those of [1], with some of the more restrictive assumptions replaced with additional symmetry and uncertainty in the recontracting process. In contrast to [3,5] and other non-cooperative models of coalition formation [8,9] which assume convexity or total balancedness of the characteristic function, the process in the paper gives convergence for any superadditive characteristic function with nonempty interior core. This result is summarized in Theorem 1.

The second theorem of this paper characterizes the stochastically stable states of the dynamic process – given the players' utility functions $u(.)$, it selects within the core allocations. For a common class of utility functions, those of the form $u(d) = ad^b$, stochastically stable states maximize a Rawlsian social welfare function: the stability of a core allocation increases in the wealth of the poorest player. Furthermore, when $u(.)/u'(.)$ is convex, a class of utilities which includes CRRA utility with preexisting wealth, the stochastically stable state is unique. When this condition holds and $u(.)$ is concave, the stochastically stable state is determined by a trade-off between maximizing the wealth of the poorest player and minimizing wealth inequality amongst the remaining players.

The learning dynamic of this paper involves groups of players jointly adjusting their strategies. Each player's strategy includes a quantity of good demanded and a set of players with whom he is willing to form coalitions. Players do not necessarily form coalitions with those with whom they discuss strategy: sometimes it is in players' best interests to agree not to be part of the same coalition. The model of this paper nests several other models. If the size of a group of players who can jointly adjust their strategies is restricted to equal one, the model is effectively that of [5]. Adding a further restriction that all players are always willing to form coalitions with all other players we get a similar model to [3]. A final restriction that gives zero value to all coalitions other than the grand coalition reduces the model to the model of Nash demand games of Young [10].

In [4] the stochastically stable states minimize the maximum weighted payoff across all players. This is because the player whose best response can change after the fewest random errors is the richest player. The presence of joint strategic switching in the current paper allows the wealth of every player to play a role in determining how vulnerable a state is to random errors and so allows a more precise characterization of stochastically stable states. However, equity considerations do still play a significant role in the selection criterion. This paper also differs from the aforementioned paper in not requiring Inada conditions on the players' utilities.

A related paper is [11] which gives stochastic stability results for coalitional recontracting in a housing economy. In the model of that paper there are multiple goods – houses – and each individual can own one and only one house. The present model has one good which can be held in any quantity. Another paper which gives core convergence is [12] which unlike this paper and the other papers in the literature relies on random errors occurring outside of the core to move the process to a core allocation.

The paper is organized as follows: Section 2 describes the stage game. Section 3 describes the learning dynamic. Section 4 is concerned with convergence to the core of the unperturbed