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A comment on: "Efficient propagation of shocks and the optimal return on money" ☆

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Abstract

Lotteries are introduced into Cavalcanti and Erosa (2008) [2], a version of Trejos and Wright (1995) [4] with aggregate shocks. Lotteries improve welfare and eliminate the two notable features of the optimum with deterministic trades: over-production and history-dependence. Moreover, the optimum can be supported by buyer take-it-or-leave-it offers.

JEL classification: C78; D61; D82; E30; E40; E50

Keywords: Random matching model of money; Aggregate shock; Optimal allocation; History-dependence; Lottery

1. Introduction

Cavalcanti and Erosa [2] (CE, hereafter) study optima in a version of Trejos and Wright [4]. They introduce into it *i.i.d.* aggregate shocks to preferences, shocks with a two-point support. They show that for an interval of intermediate magnitudes for the discount factor, the ex ante optimum over all individually rational (IR) and *deterministic* trades displays two properties: output

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☆ Cavalcanti and Erosa (2008) [2].
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is higher than the first-best when the shock is such that the first-best output is low and there is history-dependence—that is, promised utilities play a role. We show that if lotteries are allowed, then higher ex ante utility is achieved and neither property holds at an optimum.[2]

The role of lotteries in the CE setting is easily explained. Consider the situation in which the shock is such that the first-best level of output is high and in which the planner would like to weaken the seller IR constraint by making the current acquisition of money more valuable. Absent lotteries, CE achieve that by promising the current seller more output than the first-best in the future when he is a buyer and the shock is such that the first-best level of output is low.[3] With lotteries, the current acquisition of money can be made more valuable by having the buyer surrender money with some probability in that future situation.

2. Model

The model is [2] except that lotteries are allowed in trade. Time is discrete, dated as $t \geqslant 0$, and there is a unit nonatomic measure of agents. At the beginning of every period, the economy is hit by an aggregate shock s with support $\{l, h\}$, low or high, which, as described below, affects preferences. The shock s is i.i.d. over time and the probability of state s is $\pi_s (> 0)$.

Each agent maximizes the discounted sum of expected utility with discount factor $\beta \in (0, 1)$. At each date, if an agent produces $y \geqslant 0$ amount of good, the utility cost is y. If an agent consumes $y \geqslant 0$ amount of good when the current state is s, the period utility he gets is $u_s(y)$, where $u_s : \mathbb{R}_+ \to \mathbb{R}$ is differentiable, strictly increasing, strictly concave, and satisfies $u_s(0) = 0$, $u_s'(0) = \infty$ and $u_s'(\infty) = 0$. We also assume that u_s is bounded, above by $\bar{u}$, and that $u_l' < u_h'$.[4] Define the first-best output levels by $y_s^* \equiv \arg\max\{u_s(y) - y\}$ or $u_s'(y_s^*) = 1$. That is, the first-best output maximizes the sum of utilities of the consumer and the producer. It follows that $y_l^* < y_h^*$.

In each period, after the aggregate state is observed, agents are randomly matched in pairs. With probability $1/N$, an agent is a consumer, with probability $1/N$, the agent is a producer, and with probability $1 - 2/N$, the match is a no-coincidence meeting, where $N \geqslant 2$.

There exists a fixed stock of indivisible, perfectly durable money, the per capita amount of which is denoted $m \in (0, 1)$. Individual money holdings are restricted to $\{0, 1\}$. In meetings, agents' money holdings are observable, but any other information about an agent's trading history is private.

3. The planner's problem and the solution

We study the mechanism-design problem studied by CE; the planner chooses an allocation to maximize welfare subject to a notion of implementability.

The realization of the date-t aggregate shock is denoted s_t and a history up to date t is denoted $s^t \equiv (s_0, s_1, \dots, s_t)$. Let $S^t \equiv \{s_0\} \times \{l, h\}^t$ denote the set of possible histories up to date t

2 Berentsen, Molico and Wright [1] are the first to introduce lotteries into matching models of money.

3 This over-production in turn leads to history-dependence. See Proposition 10 of their paper for details.

4 One way to get the linear cost function and the bounded utility function is as follows: suppose that the utility and the cost from consuming and producing z amount are given by a possibly unbounded function $\tilde{u}_s(z)$ and a convex function $\tilde{c}(z)$, respectively. Suppose further that there is a bound $\bar{z}$ on production in a sense that $\lim_{z\to\bar{z}} \tilde{c}(z) = \infty$. Then changing the unit of goods nonlinearly by $y \equiv \tilde{c}(z)$ leads to the bounded utility function $u_s(y) \equiv \tilde{u}_s(\tilde{c}^{-1}(y))$ and the linear cost function $c(y) \equiv \tilde{c}(\tilde{c}^{-1}(y)) = y$ with no bound on y.