Available online at www.sciencedirect.com

SciVerse ScienceDirect

Journal of Economic Theory 148 (2013) 974–1049

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Ambiguity and robust statistics ☆,☆☆

Simone Cerreia-Vioglio [a], Fabio Maccheroni [a,*], Massimo Marinacci [a], Luigi Montrucchio [b]

[a] *Department of Decision Sciences, Università Bocconi, Italy*
[b] *Collegio Carlo Alberto, Università di Torino, Italy*

Received 26 April 2011; final version received 22 February 2012; accepted 8 October 2012

Available online 14 January 2013

Abstract

Starting with the seminal paper of Gilboa and Schmeidler (1989) [32] an analogy between the maxmin approach of decision theory under ambiguity and the minimax approach of robust statistics – e.g., Blum and Rosenblatt (1967) [10] – has been hinted at. The present paper formally clarifies this relation by showing the conditions under which the two approaches are actually equivalent.

JEL classification: D81

Keywords: Ambiguity; Consistency; Minimax estimation; Robust statistics; Uncertainty aversion; Variational preferences

> *Prior distributions can never be quantified or elicited exactly* (*i.e., without error*), *especially in a finite amount of time.*
>
> Berger [6, p. 64]

☆ We especially thank Rose-Anne Dana, Marcelo Moreira, Ben Polak, Kyoungwon Seo, Chris Shannon, and Piotr Zakrzewski, as well as, an associate editor and two anonymous referees for helpful suggestions. The financial support of the European Research Council (advanced grant, BRSCDP–TEA) is gratefully acknowledged.
☆☆ First presented with the title *Model Uncertainty and Sufficient Events* at RUD 2010.
* Corresponding author.
E-mail address: fabio.maccheroni@unibocconi.it (F. Maccheroni).

0022-0531/$ – see front matter © 2013 Published by Elsevier Inc.
http://dx.doi.org/10.1016/j.jet.2012.10.003

1. Introduction

Since the seminal work of Gilboa and Schmeidler [32, p. 142] a relation between decision making under ambiguity and robust statistics has been hinted at, and indeed immediate similarities are quite evident. At the same time, a formal treatment of this topic and a complete characterization of the relation between the two approaches is still missing. The object of this paper is to fill the gap, that is, relating ambiguity (also called Knightian uncertainty or model uncertainty) to prior uncertainty.

Ambiguity refers to the case in which a decision maker does not have sufficient information to quantify through a single probability distribution the stochastic nature of the problem he is facing. This uncertainty is captured by using nonadditive probabilities – capacities – or sets of probability measures over the space of states of the world (usually observations, in many economic applications).[1]

Prior uncertainty, in a parametric statistical model $\{P_\theta\}_{\theta\in\Theta}$, refers to uncertainty about the prior μ on Θ. This is a classical problem in robust statistics, where uncertainty is again represented by capacities or sets of priors over the space Θ of parameters.[2]

Clearly, prior uncertainty can be reduced to ambiguity. Loosely, if ν is a (prior) capacity on the space Θ of parameters, then

$$\bar{\nu}(\cdot) = \int_{\Theta} P_\theta(\cdot)\, d\nu(\theta) \tag{1}$$

defines a (predictive) capacity on the states of the world. Analogously, if Γ is a set of priors on the space Θ of parameters, then

$$\left\{\bar{\mu}(\cdot) = \int_{\Theta} P_\theta(\cdot)\, d\mu(\theta):\ \mu \in \Gamma\right\} \tag{2}$$

defines a set of (predictive) probability measures on the states of the world.[3]

In this paper we address the converse problem. That is, we start from a decision theoretic framework of ambiguity and we show under which conditions the decision problem admits a (suitably unique) rephrasing in terms of prior uncertainty. In this way, we are able to provide an axiomatic and behaviorally falsifiable foundation to the criteria used in robust statistics and to suggest the general form such criteria could take (see Section 4.1.1 for a discussion of the specific case of robust estimation). We achieve this goal by merging the decision theoretic assumptions that are by now established in the literature of choice under ambiguity with some of the fundamental insights contained in Wald [60], Ellsberg [24], Nehring [47], and Gilboa, Maccheroni, Marinacci, and Schmeidler [30].

We model ambiguity by using a generalized Anscombe–Aumann setting as in Gilboa, Maccheroni, Marinacci, and Schmeidler [30]. We denote by Ω the space of states of the world and

1 Early classical references are Schmeidler [53], Bewley [8], and Gilboa and Schmeidler [32]. See Gilboa and Marinacci [31] for a recent survey.

2 In the words of Blum and Rosenblatt [10, p. 1671]: "*Except in rare situations, information concerning the* a priori *distribution of a parameter is likely to be incomplete. Hence the use of a Bayes rule on some systematically produced choice for an* a priori *distribution ... is difficult to justify ... In this note we investigate ... the case in which it is known only that the distribution of the parameter is a member of some given family*." See also Randles and Hollander [49], Shafer [55], as well as Berger [6] and [7].

3 See Marinacci [45] for the distinction between prior and predictive probability measures in a setup with ambiguity.