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Illiquidity, position limits, and optimal investment for mutual funds ☆

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Abstract

We study the optimal trading strategy of mutual funds that face both position limits and differential illiquidity. We provide explicit characterization of the optimal trading strategy and conduct an extensive analytical and numerical analysis of the optimal trading strategy. We show that the optimal trading boundaries are increasing in both the lower and the upper position limits. We find that position limits can affect current trading strategy even when they are not currently binding and other seemingly intuitive trading strategies can be costly. We also examine the optimal choice of position limits.

JEL classification: D11; D91; G11; C61

Keywords: Illiquidity; Portfolio constraints; Position limits; Transaction costs; Mutual funds; Optimal investment

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1. Introduction

Mutual funds are often restricted to allocating certain percentages of fund assets to certain securities that have different degrees of illiquidity. These restrictions are often specified in a fund's prospectus and differ across investment styles and country-specific regulations. For example, a small cap fund may set a lower bound on its holdings of small cap stocks. In Switzerland, regulations require that at least two thirds of a fund's assets be invested in the relevant geographical sectors (e.g., Switzerland, Europe) or asset classes depending on the fund's category. In France, regulations prevent bond and money market funds from investing more than 10% in stocks. Mutual funds can also face significant transaction costs in trading securities in some asset classes. Wermers [33] concludes that transaction costs drag down net mutual fund returns by as much as 0.8%, about the same impact as fund expenses. Consistent with this finding, Chalmers et al. [8] conclude that annual trading costs for equity mutual funds are large and exhibit substantial cross-sectional variation, averaging 0.78% of fund assets per year and having an interquartile range of 0.59%. Karceski et al. [24] find that about 46% of all small cap mutual funds have trading costs that are higher than the annual fees investors pay. The prevalence of turnover constraints also suggests the importance of transaction costs (e.g., Clarke et al. [9]).

There is a large literature on the optimal trading strategy of a mutual fund.[1] However, most of this literature does not consider the significant trading costs faced by funds or the widespread position limits imposed upon mutual funds. As is well known, the presence of transaction costs and position limits can have a drastic impact on the optimal trading strategy and portfolio performance.[2] The coexistence of the position limits and asset illiquidity and the interactions among them can thus be important for the optimal trading strategy of a mutual fund. However, the existing literature ignores this coexistence and the interactions. Therefore, the optimal trading strategy of a fund that is subject to position limits and asset illiquidity is still unknown.

In this paper, we study the optimal investment problem of a mutual fund that faces position limits and trades a risk-free asset, a liquid stock, and an illiquid stock that is subject to proportional transaction costs. Because the implied Hamilton–Jacobi–Bellman equation is highly nonlinear and difficult to analyze, we convert the original problem into a double obstacle problem that is much easier to analyze. Using this alternative approach, we are able to characterize the value function and to provide many analytical comparative statics on the optimal trading strategy. We show that there exists a unique optimal trading strategy and the value function is smooth except on a measure zero set. The optimal trading strategy for the illiquid stock is determined by a time-varying buy boundary and a time-varying sell boundary between which no transaction occurs.

In addition, we establish some important monotonicity properties analytically for the trading boundaries, which are also useful for improving the precision and robustness of the numerical procedure. For example, both the buy boundary and the sell boundary (in terms of the fraction of assets under management (AUM) invested in the illiquid stock) are monotonically increasing in the position limits. In addition, in most cases the optimal buy (sell) boundary is monotonically decreasing (increasing) in calendar time when time to horizon is short.

We also conduct an extensive numerical analysis on optimal trading strategies. Our numerical analysis shows that in the presence of transaction costs, even for log preferences, the optimal

[1] See, for example, Carpenter [7], Basak et al. [6], Cuoco and Kaniel [13].

[2] See, for example, Davis and Norman [21], Cuoco [12], Cuoco and Liu [15], Balduzzi and Lynch [5], Liu and Loewenstein [28], Liu [27].