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Recursive smooth ambiguity preferences

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Abstract

This paper axiomatizes an intertemporal version of the Smooth Ambiguity decision model developed in [P. Klibanoff, M. Marinacci, S. Mukerji, A smooth model of decision making under ambiguity, Econometrica 73 (6) (2005) 1849–1892]. A key feature of the model is that it achieves a separation between ambiguity, identified as a characteristic of the decision maker's subjective beliefs, and ambiguity attitude, a characteristic of the decision maker's tastes. In applications one may thus specify/vary these two characteristics independent of each other, thereby facilitating richer comparative statics and modeling flexibility than possible under other models which accommodate ambiguity sensitive preferences. Another key feature is that the preferences are dynamically consistent and have a recursive representation. Therefore techniques of dynamic programming can be applied when using this model.

JEL classification: D800; D810

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1. Introduction

This paper axiomatizes and investigates a model of recursive preferences over intertemporal plans, extending the smooth ambiguity model developed in Klibanoff, Marinacci, and Mukerji [28] (henceforth KMM) to a setting involving dynamic decision making.

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In KMM we propose and axiomatize a model of preferences over acts such that the decision maker prefers act f to act g if and only if $\mathbb{E}_\mu \phi(\mathbb{E}_\pi u \circ f) \geqslant \mathbb{E}_\mu \phi(\mathbb{E}_\pi u \circ g)$, where $\mathbb{E}$ is the expectation operator, u is a vN–M utility function, ϕ is an increasing transformation, and μ is a subjective probability over the set Π of probability measures π that the decision maker thinks are relevant given his subjective information. A key feature of our model is that it achieves a separation between ambiguity, identified as a characteristic of the decision maker's subjective beliefs, and ambiguity attitude, a characteristic of the decision maker's tastes. We show that attitudes towards pure risk are characterized by the shape of u, as usual, while attitudes towards ambiguity are characterized by the shape of ϕ. Ambiguity itself is defined behaviorally and is shown to be characterized by properties of the subjective set of measures Π. One advantage of this model is that the well-developed machinery for dealing with risk attitudes can be applied as well to ambiguity attitudes. The model is also distinct from many in the literature on ambiguity in that it allows smooth, rather than kinked, indifference curves. This leads to different behavior and improved tractability, while still sharing the main features (e.g., Ellsberg's paradox). The maxmin expected utility model (e.g., Gilboa and Schmeidler [15]) with a given set of measures may be seen as a limiting case of our model with infinite ambiguity aversion.[1]

The functional representation obtained in KMM is particularly useful in economic modeling in answering comparative statics questions involving ambiguity. Take an economic model where agents' beliefs reflect some ambiguity. Next, without perturbing the information structure, it is useful to know how the equilibrium would change if the extent of ambiguity aversion were to decrease; e.g., if we were to replace ambiguity aversion with ambiguity neutrality, holding information and risk attitude fixed. (See, for example, Gollier [16] for a portfolio choice application.) Another useful comparative statics exercise is to hold ambiguity attitudes fixed and ask how the equilibrium is affected if the perceived ambiguity is varied (see Jewitt and Mukerji [24] for a definition and characterization of the notion of "more ambiguous"). Working out such comparative statics properly requires a model which allows a conceptual/parametric separation of (possibly) ambiguous beliefs and ambiguity attitude, analogous to the distinction usually made between risk and risk attitude. The model and functional representation in KMM allows that, whereas such a separation is not evident in the pioneering and most popular decision making models that incorporate ambiguity, namely, the multiple priors/maxmin expected utility (MEU) preferences (Gilboa and Schmeidler [15]) and the Choquet expected utility (CEU) model of Schmeidler [43].

While the preference model in KMM achieves the task of separating ambiguity and ambiguity attitude, the scope of application of this model is limited by the fact that it is a timeless framework. Many economic questions involving uncertain environments, especially in macroeconomics and finance, are more intuitively modeled using intertemporal decision making frameworks. It is of interest to re-examine such questions by adding an ambiguity dimension. Computation and analysis of intertemporal choices is greatly facilitated by applying recursive methods. For these methods to be applicable, preferences have to satisfy a certain dynamic consistency property. A number of recent papers, including Epstein and Schneider [10], Wang [51], Hayashi [23], and Maccheroni, Marinacci, and Rustichini [34], have provided preference foundations for extending other ambiguity models to an intertemporal framework while satisfying this dynamic consistency. All of these, however, share the limitation inherent in the atemporal models they extend, of failing to separate ambiguity from ambiguity attitude without restricting the range of

[1] For alternative developments of similar models see Ergin and Gul [13], Nau [38], Neilson [39], Seo [46]. All of these models draw inspiration from Segal [44], the earliest paper relating ambiguity sensitive behavior to a two-stage functional relaxing reduction.