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Hierarchies of ambiguous beliefs

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Abstract

We present a theory of interactive beliefs analogous to Mertens and Zamir [Formulation of Bayesian analysis for games with incomplete information, Int. J. Game Theory 14 (1985) 1–29] and Brandenburger and Dekel [Hierarchies of beliefs and common knowledge, J. Econ. Theory 59 (1993) 189–198] that allows for hierarchies of ambiguity. Each agent is allowed a compact set of beliefs at each level, rather than just a single belief as in the standard model. We propose appropriate definitions of coherency and common knowledge for our types. Common knowledge of coherency closes the model, in the sense that each type homeomorphically encodes a compact set of beliefs over the others' types. This space universally embeds every implicit type space of ambiguous beliefs in a beliefs-preserving manner. An extension to ambiguous conditional probability systems [P. Battigalli, M. Siniscalchi, Hierarchies of conditional beliefs and interactive epistemology in dynamic games, J. Econ. Theory 88 (1999) 188–230] is presented. The standard universal type space and the universal space of compact continuous possibility structures are epistemically identified as subsets.

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1. Introduction

The idea of a player's type introduced by Harsanyi [19] provides a useful and compact representation of the interactive belief structures that arise in a game, encoding a player's beliefs on some "primitive" parameter of uncertainty, her belief about the others' beliefs, their beliefs about her belief about their beliefs, and so on. Mertens and Zamir [31], hereafter MZ, constructed a universal type space encoding all internally consistent streams of beliefs, ensuring that Bayesian games with Harsanyi types lose no analytic generality.[1]

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[1] An earlier discussion of the problem can be found in [2,8].

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There remains a fundamental caveat. This notion of type implicitly assumes probabilistic sophistication, in the sense that each player has precise beliefs. In reality, the decision maker's beliefs can be ambiguous, as pointed out by Ellsberg [13], and she may consider multiple beliefs to be plausible [7,17]. Even given a precise assessment of the natural uncertainty, she may be ambiguous of her opponents' beliefs, or whether they hold precise beliefs. In games, agents may have multiple levels of multiple beliefs. A growing literature studies interactive situations with sets of beliefs [3, 20–30], for which the standard construction is inadequate.

We construct a model of interactive beliefs where each player is allowed a compact set of multiple priors. In turn, she is allowed multiple beliefs about the possibly multiple priors of the other player, and so on. If agents share common knowledge of the internal consistency of their orders of ambiguous beliefs, then an agent's type completely specifies her set of joint beliefs on the primitive state and the other's type. This space is universal in the sense that it can embed any other type space with this property in a manner that preserves the implicit hierarchies of belief. Two significant subspaces are the standard universal type space and the universal space of compact continuous possibility models [30].

The preceding criticism of the standard construction is hardly new. In fact, Epstein and Wang [15] address these concerns with hierarchies of preferences over acts. This approach has been recently extended by Di Tillio [12] for finite games. Instead of working with preferences, we explicitly model ambiguity with sets of beliefs. The comparison is clearer after formally introducing our model, hence postponed until Section 4.

2. Model

We build our model of interactive ambiguity, extending the economical construction of Brandenburger and Dekel [10], henceforth BD. Our main line of proof, establishing conditions for the Kolmogorov Extension Theorem, parallels their development and many mathematical steps are appropriately adapted. The technical contribution is mild; such adaptations are now endemic to the literature on universal spaces.

We first introduce some notation. For any metric space X, let ΔX denote its Borel probability measures endowed with the topology of weak convergence, metrized by the Prohorov distance ρ. If X is compact Polish, then ΔX is compact Polish. If Y is also metric, for any measurable $f : X \to Y$, let $\mathscr{L}_f : \Delta X \to \Delta Y$ denote the law or image measure on Y induced by f, defined by $[\mathscr{L}_f(\mu)](E) = \mu(f^{-1}(E))$ for any $\mu \in \Delta X$ and any Borel set $E \subseteq Y$. If $\mu \in \Delta(X \times Y)$, its marginal measure on X is defined as $\operatorname{marg}_X \mu = \mathscr{L}_{\mathrm{Proj}_X}(\mu)$, where Proj_X denotes projection to X.

Lemma 1. *Suppose X, Y, Z are compact metric spaces and $f : X \to Y$, $g : Y \to Z$ are measurable. Then*

1. $\mathscr{L}_{g\circ f} = \mathscr{L}_g \circ \mathscr{L}_f$;
2. *if f is continuous, then $\mathscr{L}_f$ is continuous;*
3. *if f is injective, then $\mathscr{L}_f$ is injective;*
4. *if f is surjective, then $\mathscr{L}_f$ is surjective.*

Let $\mathscr{K}(X)$ denote the family of nonempty compact subsets of X, endowed with the Hausdorff distance metric $d_h : \mathscr{K}(X) \to \mathbb{R}$:

$$d_h(A, B) = \max\left\{\max_{a\in A} \min_{b\in B} d(a, b), \max_{b\in B} \min_{a\in A} d(a, b)\right\},$$