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The communication requirements of social choice rules and supporting budget sets

Ilya Segal*

Department of Economics, Stanford University, Stanford, CA 94305-6072, USA

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Abstract

The paper examines the communication requirements of social choice rules when the (sincere) agents privately know their preferences. It shows that for a large class of choice rules, any minimally informative way to verify that a given alternative is in the choice rule is by verifying a "budget equilibrium", i.e., that the alternative is optimal to each agent within a "budget set" given to him. Therefore, *any* communication mechanism realizing the choice rule must find a supporting budget equilibrium. We characterize the class of choice rules that have this property. Furthermore, for any rule from the class, we characterize the minimally informative messages (budget equilibria) verifying it. This characterization is used to identify the amount of communication needed to realize a choice rule, measured with the number of transmitted bits or real variables. Applications include efficiency in convex economies, exact or approximate surplus maximization in combinatorial auctions, the core in indivisible-good economies, and stable many-to-one matchings.

JEL classification: D71; D82; D83

Keywords: Communication complexity; Message space dimension; Informational efficiency; Nondeterministic communication (verification); Price mechanisms; Social choice rules; Pareto efficiency; Stability; Envy-free allocations; Approximation; Convex economies; Indivisible goods; Combinatorial auctions; Stable many-to-one matching

1. Introduction

This paper considers the problem of finding allocations that satisfy certain social goals when economic agents have private information regarding their preferences. This problem has received renewed interest in the literature on "market design"—in particular, in two-sided matching (e.g., [44]) and combinatorial auctions (e.g., [5]). The goals of market design include exact or

* Fax: +1 650 725 5702.
E-mail address: ilya.segal@stanford.edu.

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approximate efficiency, voluntary participation, stability to group deviations, and some notions of fairness. A key theme in the literature is that incentives alone do not determine the choice of the mechanism. Indeed, if incentive compatibility were the only concern, it could be ensured with a direct revelation mechanism. However, full revelation of agents' preferences is often impractical, for several reasons. First, sometimes full revelation would require a prohibitive amount of communication: for example, in a combinatorial auction, a bidder would have to communicate his valuations for all possible bundles of objects, and the number of such bundles grows exponentially with the number of objects. Second, agents may have an "evaluation cost" of learning their own preferences. Finally, agents may desire to keep some preference information private (e.g., because they are worried about possible abuse of this information by the designer or other agents in future interactions).

For all these reasons, the "market design" literature has proposed a variety of mechanisms that achieve the desired goals without fully revealing agents' preferences. This raises the question: What is the minimal information that must be elicited by the designer in order to achieve the goals (even if agents are sincere)?

The problem of communication in economic mechanisms was first discussed by Hayek [19], who called attention to the "problem of the utilization of knowledge that is not given to anyone in its totality," when "practically every individual... possesses unique information of which beneficial use might be made." Hayek argued that "we cannot expect that this problem will be solved by first communicating all this knowledge to a central board which, after integrating all knowledge, issues its orders." Instead, "the ultimate decisions must be left to the people who are familiar with the... particular circumstances of time and place." At the same time, the decisions must be guided by prices, which summarize the information needed "to co-ordinate the separate actions of different people." While Hayek did not discuss allocation mechanisms other than the price mechanism and central planning (full revelation), he noted that "nobody has yet succeeded in designing an alternative system" that would fully utilize individual knowledge.

While price mechanisms have received extensive scrutiny since Hayek, existing research has failed to answer the following questions:

(1) Is it ever *necessary* to find some supporting prices in order to achieve social goals?
(2) For which *preference domains* is it necessary to find supporting prices?
(3) For which *social goals* is it necessary to find supporting prices?
(4) *What kind of prices* verify a given social goal on a given preference domain while revealing the minimal necessary information?

Economists have often justified the use of price mechanisms with the Fundamental Theorems of Welfare Economics. However, these theorems fail to address even question (1). Indeed, the First Welfare Theorem says that supporting prices are sufficient to verify Pareto efficiency, but does not establish their necessity. The Second Welfare Theorem only says that supporting prices can be constructed for a given Pareto efficient allocation once all the information about the economy is available. However, once all the information is available, the desired allocation can be imposed directly, without using prices. The theorems have nothing to say about possible efficient nonprice mechanisms in an economy with distributed knowledge of preferences.

Similarly, economists have emphasized the interpretation of prices as the dual variables (Lagrange multipliers) for an optimization program. (Just as the Second Welfare Theorem, this interpretation is based on the Separating Hyperplane Theorem.) However, there are many optimization algorithms that do not use dual variables and do not find their equilibrium values (e.g., the simplex