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A subjective model of experimentation

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Abstract

Typically an experiment is defined by a set of possible signals and a likelihood function, and both are specified exogenously—they are taken to be observable by the analyst. This paper renders them subjective by showing that they may be derived from suitable choice behavior. This is done in the context of an axiomatic representation theorem for preference on a suitable domain.

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1. Introduction

1.1. Objective

There are many situations where we take an action in order to experiment. Since the realization of signals depends on the action, the agent can decide how much information to acquire by varying actions.

Consider the standard model of experimentation. Let Ω be a (finite) set of states of the world. The true state $\omega \in \Omega$ is unknown to the agent and she is assumed to have a prior $p \in \Delta(\Omega)$.[2] The agent's action $a \in \mathcal{A}$ affects the realization of signals. This relation is summarized by a function $l : \Omega \times \mathcal{A} \to \Delta(Y)$, where Y is a set of signals. For every action a, $l(y|\omega, a)$ denotes the probability of observing signal y conditional on the true state being ω. Each action is interpreted

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[2] For any space Z, $\Delta(Z)$ denotes the set of all Borel probability measures over Z.

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as an experiment. The pair $(Y, l(\cdot|\cdot, a))$ is referred to as an experiment. Given a choice of an action and hence choice of an experiment, the agent learns about the true state by observing a signal and then computing a posterior according to Bayes' Rule:

$$p(\omega|y, a) = \frac{l(y|\omega, a)p(\omega)}{\sum_{\omega' \in \Omega} l(y|\omega', a)p(\omega')}. \tag{1}$$

She chooses an alternative from a choice set according to this posterior and maximizes expected utility.

In the standard model, the pair (Y, l) is taken as a primitive. Thus it is assumed observable by the modeler. However, just as the prior p is not observable, l may also be unobservable. For example, consider a manager who seeks a worker and gives candidates a test in order to get information about their abilities. Then different managers may expect different relations between the abilities of candidates (the true state) and the result of the test (the signal). And the relations are not observable by others. Moreover, in reality, signals Y can hardly be simple. It is difficult for the modeler to know which signal the agent observes. For example, if the manager chooses a computer-scored exam as the test, then signals are simple—they are just scores. In contrast, if she chooses an interview as the test, then the signals are difficult for the modeler to define. Thus the pair (Y, l) is arguably unobservable. Just as Savage [10] makes the prior p subjective, the goal of our model is to make the pair (Y, l), in addition to the prior, subjective.

In our model, the agent has a preference over pairs of actions and menus (sets) of Anscombe–Aumann acts, $h : \Omega \to \Delta(C)$, where C is a set of consumptions. There are two periods—an ex ante period 0 when the agent chooses an action a and a menu x, and an ex post period 1 when a signal is received. The action a randomly generates a signal. In period 1, she observes the signal, updates her belief about ω, and chooses an act h from the menu x. Since she is forward-looking, her ex ante choice of an action-menu pair reflects her view of the relation between signals and actions. Therefore preference over pairs of actions and menus reveals the signal space and the action-conditional likelihoods attached to signals.

We axiomatize preference over action-menu pairs, such that a pair (a, x) is evaluated by

$$U(a, x) = W\left(v(a), \int_Y \left[\max_{h \in x} \sum_{\Omega} u(h(\omega))p(\omega|y, a)\right] p(dy|a)\right), \tag{2}$$

where $p(\omega|y, a)$ satisfies Bayes' Rule (1) and $p(y|a)$ is an ex ante probability of signal y conditional on action a, that is,

$$p(y|a) = \sum_{\omega \in \Omega} l(y|\omega, a)p(\omega).$$

Our representation (2) has two components. First, an action a has direct utility $v(a)$. Second, it has the indirect effect on the value of menu x through the change of likelihood $l(\cdot|\cdot, a)$ and hence change of ex ante probability $p(\cdot|a)$ of signals. These are aggregated by an aggregator W. To understand the indirect effect, work backward. Suppose that the agent receives a signal y after choosing (a, x). Then she chooses the best act h from the menu x according to the posterior $p(\omega|y, a)$. The maximized utility, $\max_{h \in x} \sum_{\Omega} u(h(\omega))p(\omega|y, a)$, depends on the signal. Therefore, in period 0, the value of menu x is the expected value of the maximized utility according to the ex ante probability $p(y|a)$. Thus the action a determines the value of menu x through its effect on the likelihood $l(\cdot|\cdot, a)$. Most standard models (e.g., [1]) assume that the aggregator is additive,