Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 145 (2010) 1757–1775
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Simplicity and likelihood: An axiomatic approach ☆

Itzhak Gilboa [a,b,*], David Schmeidler [b,c]

[a] *HEC, Paris, France*
[b] *Tel-Aviv University, Israel*
[c] *The Ohio State University, United States*

Received 2 October 2008; final version received 3 September 2009; accepted 19 February 2010

Available online 27 March 2010

Abstract

We suggest a model in which theories are ranked given various databases. Certain axioms on such rankings imply a numerical representation that is the sum of the log-likelihood of the theory and a fixed number for each theory, which may be interpreted as a measure of its complexity. This additive combination of log-likelihood and a measure of complexity generalizes both the Akaike Information Criterion and the Minimum Description Length criterion, which are well known in statistics and in machine learning, respectively. The axiomatic approach is suggested as a way to analyze such theory-selection criteria and judge their reasonability based on finite databases.

JEL classification: C1; D8

Keywords: Maximum likelihood; Simplicity; Model selection; Akaike Information Criterion; Minimum Description Length; Axioms

1. Introduction

The selection of a theory based on observations is a fundamental problem that cuts across several disciplines. Finding the "right" way to select theories given evidence is at the heart of

☆ We thank Yoav Binyamini, Offer Lieberman, two anonymous referees and the associate editor for comments and suggestions. This project was supported by the Pinhas Sapir Center for Development and Israel Science Foundation Grant Nos. 975/03 and 355/06.

* Corresponding author at: Tel-Aviv University, Israel.
E-mail addresses: tzachigilboa@gmail.com (I. Gilboa), schmeid@tau.ac.il (D. Schmeidler).

0022-0531/$ – see front matter
doi:10.1016/j.jet.2010.03.010

philosophy of science, statistics, and machine learning. It is also highly relevant to rational models of learning, trying to capture the way that rational agents can make sense of the data available to them.

Two fundamental criteria for the selection of theories are simplicity and goodness of fit. The preference for simple theories is well known, and is often attributed to William of Occam (see Russell [11]). While the notion of simplicity is partly subjective and depends on language,[1] it is often surprising how much agreement one finds between the simplicity judgment of different people. For example, most people tend to agree that, other things being equal, a theory with fewer parameters is simpler than a theory with more parameters, or that a theory with a shorter description is simpler than a theory with a longer one. Whereas such claims depend on an agreement about a language, or a set of languages within which simplicity is measured, they do not seem to be vacuous statements. The suggestion that people tend to prefer simple theories to more complex ones can therefore be a meaningful empirical claim.

However, simplicity can only serve as an a priori argument for or against certain theories. How well a theory performs in explaining observed data should certainly also factor into our considerations in selecting theories. Sometimes, one may categorize theories dichotomously into theories that fit the data as opposed to theories that are refuted by the data, and then choose the simplest theory among the former. But in most problems in science, as well as in everyday life, theories are never categorically refuted. There is typically room for a measurement error, or, more generally, for probabilistic prediction. Therefore, a theory typically cannot be refuted by observations. Instead, theories can be ranked according to their goodness of fit, namely, the extent to which they match observations. In particular, the likelihood principle suggests to rank theories according to their likelihood function, that is, the a priori probability that the theory used to assign to the observed data before these data were indeed observed.

Viewed from a statistical point of view, the likelihood principle is a fundamental idea that neatly captures the notion of "goodness of fit" while relying on objective data alone. Choosing the theory that maximizes the conditional probability of the actually observed sample does not rely on any subjective a priori preferences, hunches, or intuitions of the reasoner. But for that very reason, the maximum likelihood principle cannot express preferences for simplicity. Due to this limitation, the applicability of this criterion is restricted to set-ups in which the set of possible theories is restricted a priori to a given class, within which complexity considerations might be ignored. When no such a priori restriction is available, the maximum likelihood principle is insufficient. More explicitly, if one considers all conceivable theories, one will always be able to find a theory that matches the observations perfectly. Such a theory will obtain the maximum conceivable likelihood value of 1, but it is likely to be "overfitting" the data. We tend not to trust a theory that matches the data perfectly if it appears very complex. Thus, maximum likelihood does not suffice to describe the totality of considerations that enter the theory selection process.[2]

We are therefore led to the conclusion that a reasonable criterion for the selection of theories based on observations has to take into account both the likelihood of a theory, or some other measure of goodness of fit, and its simplicity, or some other a priori preference for some theories versus others.[3] Indeed, combinations of likelihood and some measure of complexity are well

[1] See Goodman [4] and Sober [14].

[2] See Gilboa and Samuelson [6] who suggest an evolutionary argument for the preference for simplicity.

[3] Another relevant criterion is the theory's generality. In this paper we ignore the more involved three-way trade-off between goodness of fit, simplicity, and generality. We will only mention in passing that if we "normalize" the theories under comparison so that they have the same scope of applicability, preference for generality can be derived