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The rate of convergence to perfect competition of matching and bargaining mechanisms ☆

Artyom Shneyerov [a,*], Adam Chi Leung Wong [b]

[a] *CIREQ, CIRANO and Department of Economics, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal, Quebec H3G 1M8, Canada*
[b] *School of International Business Administration, Shanghai University of Finance and Economics, 777 Guoding Road, Shanghai, China 200433*

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Abstract

We study the steady state of a market with incoming cohorts of buyers and sellers who are matched pairwise and bargain under private information. A friction parameter is τ, the length of the time period until the next meeting. We provide a necessary and sufficient condition for the convergence of mechanism outcomes to perfect competition at the linear rate in τ, which is shown to be the fastest possible among all bargaining mechanisms. The condition requires that buyers and sellers always retain some bargaining power. The bargaining mechanisms that satisfy this condition are called nonvanishing bargaining power (NBP) mechanisms. Simple random proposer take-it-or-leave-it protocols are NBP, while k-double auctions (k-DA) are not. We find that k-DAs have equilibria that converge to perfect competition at a linear rate, converge at a slower rate or even do not converge at all.

JEL classification: C73; C78; D83

Keywords: Matching and bargaining; Search; Double auctions; Foundations for perfect competition; Rate of convergence

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* Corresponding author. Fax: +1 514 848 4536.
E-mail addresses: achneero@alcor.concordia.ca (A. Shneyerov), wongchileung@gmail.com (A.C.L. Wong).

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1. Introduction

A number of papers on dynamic matching and bargaining have shown that, as frictions vanish, equilibria converge to perfect competition.[1] But it is also important to know how rapidly the equilibria converge. To our knowledge, this question has not been addressed in the literature.

In contrast, the rate of convergence to efficiency has been the focus of the literature on static double auctions. It is important to know how large n needs to be so that we can call a double auction with n buyers and sellers approximately competitive. Rustichini, Satterthwaite, and Williams [23] show robust convergence of double-auction equilibria in the symmetric class at the fast rate $O(1/n)$ for the bid/offer strategies and the superfast rate $O(1/n^2)$ for the ex ante traders' welfare, where n is the number of traders in the market.[2] Moreover, the double auction converges at the rate that is fastest among all incentive-compatible and individually rational mechanisms (see [28, 31]). Cripps and Swinkels [7] substantially enrich the model by allowing correlation among bidders' valuations, and show convergence at the rate $O(1/n^{2-\varepsilon})$, where $\varepsilon > 0$ is arbitrarily small.[3]

For a dynamic matching and bargaining market, the question of how small frictions need to be for equilibria to be approximately competitive is equally important. In this paper, we fill this gap by proving a rate of convergence result for a decentralized model of trade. We study the steady state of a market with incoming cohorts of buyers and sellers who are randomly matched pairwise and bargain without knowing each other's reservation value. The model is in discrete time and shares several features with the model in [25]. Exactly as in that paper, a friction parameter is τ, the length of the time period until the next meeting. There are per-period participation costs, $\tau\kappa_B$ for buyers and $\tau\kappa_S$ for sellers. There is also time discounting at the instantaneous rate r.[4]

Our model is different from Satterthwaite and Shneyerov [25] in that we consider pairwise matching and general trading mechanisms. (Satterthwaite and Shneyerov [25] restrict attention to auctions.) Atakan [2] provides an important extension of the results of Satterthwaite and Shneyerov [25] to multiple units and allows each trader to be a proposer with certain probability. Atakan [2] allows the proposers to offer direct bargaining mechanisms (DBMs, as defined in [18]), and shows that in equilibrium they can do no better than simply make price offers.[5] With this justification, he confines the analysis to take it or leave it price offer games.[6]

We consider a class of DBMs, *nonvanishing bargaining power* (*NBP*) *mechanisms*, that generalize certain properties of the random-proposer take it or leave it games described above. As the name suggests, the NBP conditions require that each trader has at least some bargaining power even when $\tau \to 0$. When a buyer with valuation v meets a seller with cost c, there is an expected bargaining surplus $U(v, c)$ available to them, over and above their market values of search. For

[1] Papers that address convergence include Rubinstein and Wolinsky [21], Gale [10], Gale [11], Rubinstein and Wolinsky [22], Mortensen and Wright [17], and, with private information, Butters [5], Wolinsky [33], De Fraja and Sakovics [9], Serrano [29], Moreno and Wooders [16], Lauermann [13], Satterthwaite and Shneyerov [25], Atakan [2].

[2] See also [12,24,27,32].

[3] Reny and Perry [19] allow interdependent values and show that it is almost efficient and almost fully aggregates information as $n \to \infty$, but do not address the rate of convergence issue.

[4] Satterthwaite and Shneyerov [25] also discuss the rate of convergence to perfect competition, and the relevance of making τ small. However, there is no general rate of convergence result, but only within a class of full trade equilibria. They also discuss an interpretation of the inverse of τ as a measure of local market size. We do not repeat these discussions here.

[5] This parallels the no haggling result in [20].

[6] Shneyerov and Wong [30] derive necessary and sufficient conditions for the existence of nontrivial market equilibria for this protocol.