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Aggregation of binary evaluations ☆

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Abstract

We study a general aggregation problem in which a society has to determine its position (yes/no) on each of several issues, based on the positions of the members of the society on those issues. There is a prescribed set of feasible evaluations, i.e., permissible combinations of positions on the issues. This framework for the theory of aggregation was introduced by Wilson and further developed by Rubinstein and Fishburn. Among other things, it admits the modeling of preference aggregation (where the issues are pairwise comparisons and feasibility reflects rationality), and of judgment aggregation (where the issues are propositions and feasibility reflects logical consistency). We characterize those sets of feasible evaluations for which the natural analogue of Arrow's impossibility theorem holds true in this framework.

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1. Introduction

Various problems of aggregation may be cast in the following framework. A society has to determine its positions on each of several issues. There are two possible positions (say, 0 or 1) on

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each issue, but the issues are interrelated and therefore not all combinations of 0–1 positions are feasible. Some set X of 0–1 vectors (of length equal to the number of issues, which we assume to be finite) is given, representing the feasible combinations of positions for each individual in the society as well as for the society as a whole. An aggregator is a function that assigns to every possible profile of individual evaluations in the set X, a social evaluation in the set X. The question is, how do well-behaved aggregators (i.e., that satisfy certain natural conditions) look like, and in particular, under what conditions are we forced to use dictatorial aggregators.

The first to propose such a framework for aggregation theory was Wilson [17]. His motivation was to show that Arrow's [1] impossibility theorem on aggregation of preferences extends to the aggregation of attributes other than preferences. To see how the aggregation of preferences fits into the above framework, consider the case of strict preferences over three alternatives a, b, and c. In this case there are three issues: whether a is preferred to b, whether b is preferred to c, and whether a is preferred to c. Any strict preference over $\{a, b, c\}$ can be encoded as a triple of 0–1 (no/yes) answers to these three questions, but not every such triple is allowed. Transitivity of preferences rules out the triples $(0, 0, 1)$ and $(1, 1, 0)$. The set X consists of the remaining six triples, and its members correspond to the six possible strict orderings of the set $\{a, b, c\}$. An aggregator mapping profiles of triples in X to triples in X thus corresponds to a social welfare function (in which both individual and social preferences are assumed to be strict).[2]

Following Wilson, we adapt Arrow's conditions for social welfare functions to the general framework. We say that an aggregator is independent of irrelevant alternatives (abbreviated IIA) if the society's position on any given issue depends only on the individuals' positions on that same issue. We say that an aggregator is Paretian if the society adopts any unanimously held position. In Arrow's context, his impossibility theorem asserts that when there are at least three alternatives, any IIA and Paretian aggregator must be dictatorial. The main question that we study here is: in the general framework, for which sets X is it the case that every IIA and Paretian aggregator mapping profiles of evaluations in X to evaluations in X must be dictatorial. In other words, we want to identify which limitations on feasibility have the same negative implications for IIA and Paretian aggregation that transitivity has in the context of aggregating preferences.

Rubinstein and Fishburn [8,16] pursued the study of Wilson's framework, and introduced an algebraic point of view.[3] They suggested several examples of aggregation problems that fit into this framework. One such example is the aggregation of equivalence relations, as a classification tool. There is a population of items, say plants, that is to be partitioned into families of similar items according to some criteria. In this application, every pair of items forms an issue, with the entry 1 meaning that they are equivalent and 0 meaning that they are not. The set X consists of those 0–1 vectors that represent equivalence relations. The individual equivalence relations that are to be aggregated may correspond to different experts, or to different criteria of classification.

Our main result is a characterization of those subsets X of $\{0, 1\}^m$ having the property that every IIA and Paretian aggregator over X (for a society of any size) must be dictatorial. There are

[2] The original setting of Arrow's theorem allows for indifference between alternatives. This does not fit into Wilson's framework described here, and therefore we do not recover Arrow's theorem as such. Rather, we recover and generalize the variant of Arrow's theorem (often referred to in the literature also as Arrow's theorem) in which only strict preferences are allowed. In a subsequent paper [6] we extend the framework to allow for abstentions on some of the issues, and thereby recover Arrow's theorem for weak preferences.

[3] They took X to be a subset of a finite-dimensional vector space over some field. Wilson's framework (and ours) corresponds to the case when the field is the two-element field $\{0, 1\}$. Some of Rubinstein and Fishburn's treatment was also specific to this case.