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Majority voting on restricted domains ☆

Franz Dietrich [a,b], Christian List [a,*]

[a] *London School of Economics, United Kingdom*
[b] *University of Maastricht, Netherlands*

Abstract

In judgment aggregation, unlike preference aggregation, not much is known about domain restrictions that guarantee consistent majority outcomes. We introduce several conditions on individual judgments sufficient for consistent majority judgments. Some are based on global orders of propositions or individuals, others on local orders, still others not on orders at all. Some generalize classic social-choice-theoretic domain conditions, others have no counterpart. Our most general condition generalizes Sen's triplewise value-restriction, itself the most general classic condition. We also prove a new characterization theorem: for a large class of domains, if there exists any aggregation function satisfying some democratic conditions, then majority voting is the unique such function. Taken together, our results support the robustness of majority rule.
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1. Introduction

In the theory of preference aggregation, it is well known that majority voting on pairs of alternatives may generate inconsistent (i.e., cyclical) majority preferences even when all individuals'

preferences are consistent (i.e., acyclical). The most famous example is Condorcet's paradox. Here one individual prefers x to y to z, a second y to z to x, and a third z to x to y, and thus there are majorities for x against y, for y against z, and for z against x, a 'cycle'. But it is equally well known that if individual preferences fall into a suitably restricted domain, majority cycles can be avoided (see Gaertner [14] for an overview). The most famous domain restriction with this effect is Black's single-peakedness [1]. A profile of individual preferences is single-peaked if the alternatives can be ordered from 'left' to 'right' such that each individual has a most preferred alternative with decreasing preference for other alternatives as we move away from it in either direction. Inada [17] showed that another condition called single-cavedness and interpretable as the mirror image of single-peakedness also suffices for avoiding majority cycles: a profile is single-caved if, for some left–right order of the alternatives, each individual has a least preferred alternative with increasing preference for other alternatives as we move away from it in either direction. Sen [38] introduced a very general domain condition, called triplewise value-restriction, that guarantees acyclical majority preferences and is implied by Black's, Inada's and other conditions; it therefore unifies several domain-restriction conditions, yet has a technical flavor without straightforward interpretation.

The wealth of domain-restriction conditions for avoiding majority cycles was supplemented by another family of conditions based not on left–right orders of the alternatives, but on left–right orders of the individuals. Important conditions in this family are Grandmont's intermediateness [16] and Rothstein's order restriction [34,35] with its special case of single-crossingness (e.g., Roberts [32], Saporiti and Tohmé [36], Saporiti [37]). To illustrate, a profile of individual preferences is order-restricted if the individuals – rather than the alternatives – can be ordered from left to right such that, for each pair of alternatives x and y, the individuals preferring x to y are either all to the left, or all the right, of those preferring y to x.

In the theory of judgment aggregation, by contrast, domain restrictions have received much less attention (the only exception is the work on unidimensional alignment, e.g., List [22]). This is an important gap in the literature since, here too, majority voting with unrestricted but consistent individual inputs may generate inconsistent collective outputs, while on a suitably restricted domain such inconsistencies can be avoided. As illustrated by the much-discussed discursive paradox (e.g., Pettit [31]), if one individual judges that a, $a \rightarrow b$ and b, a second that a, but not $a \rightarrow b$ and not b, and a third that $a \rightarrow b$, but not a and not b, there are majorities for a, for $a \rightarrow b$ and yet for not b, an inconsistency. But if no individual rejects $a \rightarrow b$, for example, this problem can never arise.

Surprisingly, however, despite the abundance of impossibility results generalizing the discursive paradox as reviewed below, very little is known about the domains of individual judgments on which discursive paradoxes can occur (as opposed to agendas of propositions susceptible to such problems, which have been extensively characterized in the literature). If we can find compelling domain restrictions to ensure majority-consistency, this allows us to refine and possibly ameliorate the lessons of the discursive paradox. Going beyond the standard impossibility results, which all assume an unrestricted domain, we can then ask: in what political and economic contexts do the identified domain restrictions hold, so that majority voting becomes safe, and in what contexts are they violated, so that majority voting becomes problematic?

This paper has two goals. The first is to introduce several conditions on profiles of individual judgments that guarantee consistent majority judgments. These can be distinguished in at least two respects: first, in terms of whether they are based on orders of propositions, on orders of individuals, or not on orders at all; and second, if they are based on orders, in terms of whether